

ELLIPSES AND OVALS IN THE PHYSICAL SPACE OF ST. PETER'S SQUARE IN ROME

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Abstract. We discuss the spatial and perceptive implications of the geometry at urban scale of St. Peter's square in Rome. The case study of Bernini's project for the colonnade makes explicit the relation between the geometry of the plan and the spatial development. We will show that the geometry of the circle affects intentionally the perception of the walking visitor. There is an ambiguity of use between the terms ellipse and oval, especially in touristic literature and some school textbooks. The specific mathematical properties, of one curve respect to the other, were well known at the time and were obviously intentionally exploited in the design of the square along with the colonnade. We also report of a "geometric flash mob", we organized in occasion of the 2015 European Researcher's Night.

Keywords: ovals, ellipses, Bernini, spatial development, flash mob

Mathematics Subject Classification: 51M15, 00A67

1 Introduction

In this paper, we aim to investigate the 3d spatial implications, in architecture and at urban scale, of some curved geometries, giving an occasion of active citizenship and teaching. In particular, we will render explicit the relation between the geometry of the plan and the its development in the three dimensions.

The case study of Bernini's project for the colonnade of St Peter's square can be used to show the geometric differences between the ellipse and the ovate through the actual experience in the physical space. For this purpose, we will show some of the many meanings that the word elliptic has in different mathematical frameworks and describe the geometric features of the ovate.

In the architectural culture this difference has often been subject of interest because this kind of curved shapes were used for the design of buildings, construction elements or urban spaces. Some well-known examples are the ancient amphitheatres, the plan of churches, the base curve of domes, the design of low arches [17; 20].



Fig. 1. St Peter's square. Designed by G. L. Bernini (1656)

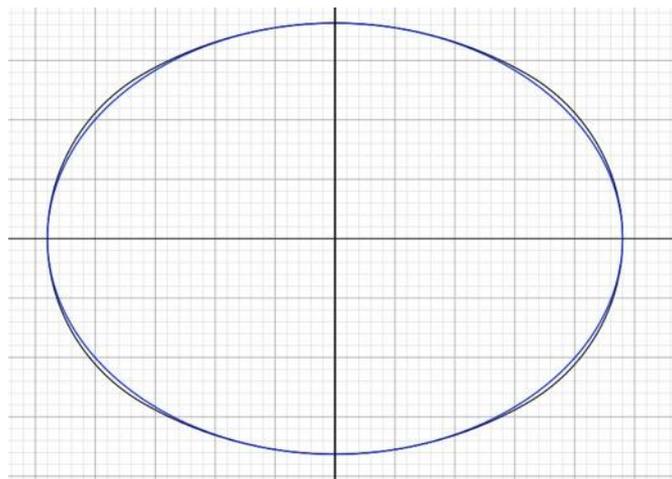


Fig. 2. An ellipse (blue), an ovate (black), inscribed in the same rectangle. The proportions are the same of those of the internal dimensions of the square. The shape of the two plots is very similar

The literature that has been produced follows two main directions: the analytical investigation [22,23] that aims to measure how much ovals constructed with arcs of circles can be similar to ellipses; the comparison between the original drawings and the relief of the state of fact [6,10,12], focusing on the analysis of the two-dimensional plan.

We propose to observe the geometry of St Peter's square from another point of view, taking the plan along with the elevation and moving in the three-dimensional space of the city. We will show that the drawing of the colonnade influence the visitor's observation who moves on foot in the square. Moreover, that the perceptive analysis of the spatial phenomenon makes explicit the geometry of the construction.

This paper is divided as follows. Section 2 is devoted to show some of the many aspects that the word elliptic addresses in mathematical frameworks, and the geometric features of the ovate.

In Section 3 we describe examples of the use, sometimes ambiguous, of the terms oval and elliptic taken from historical treatises, contemporary articles, touristic guides and schoolbooks. In Section 4 we describe the shape of St. Peter's square, and Section 5 is devoted to a description of the plan together with the elevation, in order to explain how the geometry of the circle influences the perception of the visitor who moves on foot in the square. Finally, in Section 6 we report of a flash mob organized in St Peter's square.

2 Ellipse, elliptic and oval

The ellipse is a conic section, a geometric locus, draws the orbit of planets. It has a primary role in the background of mathematicians, so that the mathematics community keeps using the name "elliptic" to describe new concepts. In this section, we want to show some of these aspects.

On the other hand, every architect knows exactly what ovate means, this word is very common in architecture, while it is not in mathematical culture.

The ellipse is a conic curve, i.e. can be obtained as a plane section of a cone (a cone is the surface generated by the rotation of a line r around another line l , being the two lines incident; the line r and all its rotated copies will be called generatrices). When the plane intersects all the generatrices (fig 1 left), the section is an ellipse.

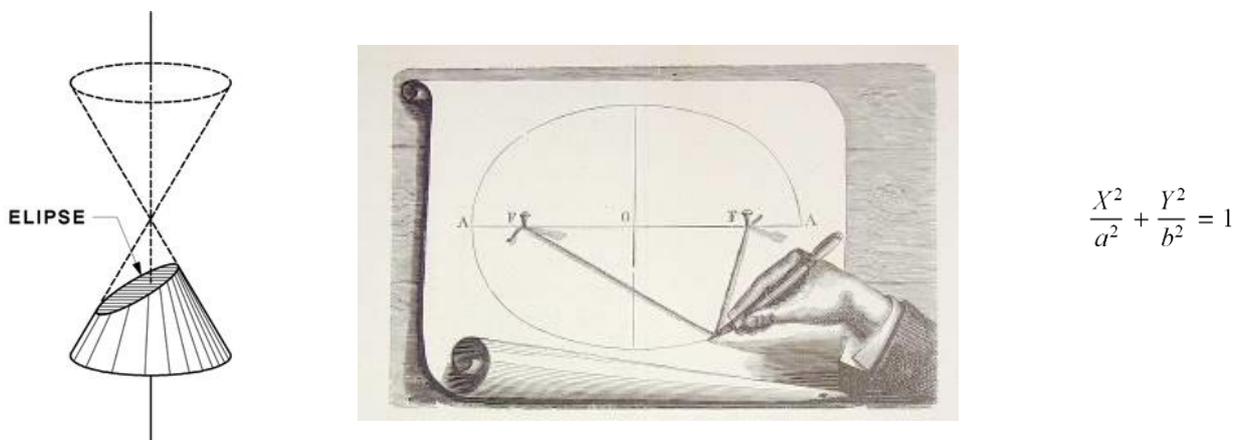


Fig. 3. From left to right: the ellipse as a plane section of the cone, the ellipse as geometric locus and its equation. Central picture: method to track an ellipse, called "the gardener's method".

Illustration from da A. Guillemin (1882), *El mundo físico: gravedad, gravitación, luz, calor, electricidad, magnetismo, etc.*

Apollonius (who lived between 262 and 190 B.C.) in his texts about conics sections writes down the equations in this way [18]:

$Y^2=Lx$ for the parabola, $Y^2=Lx+ L/aX^2$ for the hyperbola, $Y^2= Lx- L/aX^2$ for the ellipse (L is a parameter, a is a transverse axis of the ellipse, see also [42]).

One possible interpretation for the name ellipse comes from the equation, since the word derives from the greek *elleipsis* which means lack or shortage, and in the equation for the ellipse, the term L/aX^2 is subtracted.

A conic section can also be defined as the projection of a circle on a plane. So, the ellipse, which is one of the possible projections, inherits all the property of the circle that are invariant under projective transformations, such for example the cross ratio.

One of the most common definition of ellipse is that of geometric locus, "*the ellipse is a plane curve such that the sum of the distances of each point from two fixed points called foci is constant*". The mathematical machine in figure 3 allows to plot an ellipse with a continuous track and embodies the definition of the curve as geometric locus. The two pins are the foci, the inextensible rope obliges the sum of the distances to remain constant.

What is really striking is that a curve that was known since two thousand years, reveals to be the orbit of planets around the Sun. In his *De Revolutionibus*, Copernicus (1473-1543) when describing the heliocentric system speaks about orbits that are not perfectly circular, but does not mention explicitly the ellipse. It was Kepler (1571-1630) who asserted that the orbit of planets around the sun are perfect ellipses, and that the sun is in one of the foci. More in general the conic curves are three different solutions of the differential equation of the two body problem, which involves the motion of two objects, supposed to be point-like, under the action of their gravitational fields.

The measure of the length of an arc of ellipse leads to an integral, that is called elliptic, in particular of the kind

$$L(\theta) = \int_0^\theta \sqrt{1 - k^2(\sin t)^2} dt$$

almost impossible to be solved. The so called *elliptic functions* are related to the inversion of another kind of elliptic integral, not related to the computation of the arc length of an ellipse, but anyway called in this way. They were created by Abel, who had to deal with an elliptic function when aimed to divide Bernoulli's Lemniscate in n equal parts by ruler and compass [21]. The theory of elliptic functions and that of elliptic curves, the geometric counterpart, is today a lively field of research, and played an important role in the proof of Fermat's last theorem.

The equation of a conic curve, a second degree polynomial, can be associated to the 2x2 matrix, called the quadratic invariant, composed by the coefficients of the second degree terms. The determinant of the quadratic invariant allows to distinguish between the three different conic curves. In other words only the second degree coefficients determine which kind of conic is represented by the equation. When the conic is an ellipse, the determinant is strictly positive.

Transferring the information on the higher degree terms to differential operators, elliptic refers to a particular class among them. A differential operator is called uniformly elliptic if coefficients of the highest-order derivatives are positive. These operators are a generalization of the Laplace operator (actually of the negative Laplacian) and elliptic equations are the partial differential equations related to elliptic operators.

In the framework of non-Euclidean geometry, the elliptic or Riemannian geometry is the one that generalizes the geometry on the sphere. It was introduced by Riemann in the inaugural speech of 1851 at Gottingen University. It is a geometry where lines are not infinite, but finite and closed [8], in particular, on a sphere, lines are great circles. In a more general example of Riemannian geometry, there can be points of two different kind. Elliptic points: in the neighborhood of the point the surface is like a sphere, the tangent plane can be shifted towards the surface and cuts the surface in a curve that looks like an ellipse. Hyperbolic points, if the tangent plane is shifted towards the surface, the intersection is similar to a hyperbola.

Finally, if it is possible to calculate the Gaussian curvature of the surface at a point, elliptic points are those where this curvature is strictly positive, which, in terms of differential calculus means that the homogeneous second order derivatives have the same sign.

We will now briefly discuss the definition of the term ovate. In architectural framework, the word oval or ovate refers to a particular polycentric curve, that is a curve formed by arcs of circles. As it will be showed in section 3, polycentric curves, in particular ovals, were created for practical purposes and widely used in architecture.

A rigorous definition of oval can be: *“an oval is a closed convex curve, continuous and differentiable”*. So, an ellipse can be an oval, since it agrees with the definition above. A polycentric curve is defined as: *“line formed by several arcs of circumference, mostly of different radii, which, at the connection points, have the same tangent line”* (cit. Treccani Enciclopedia online).

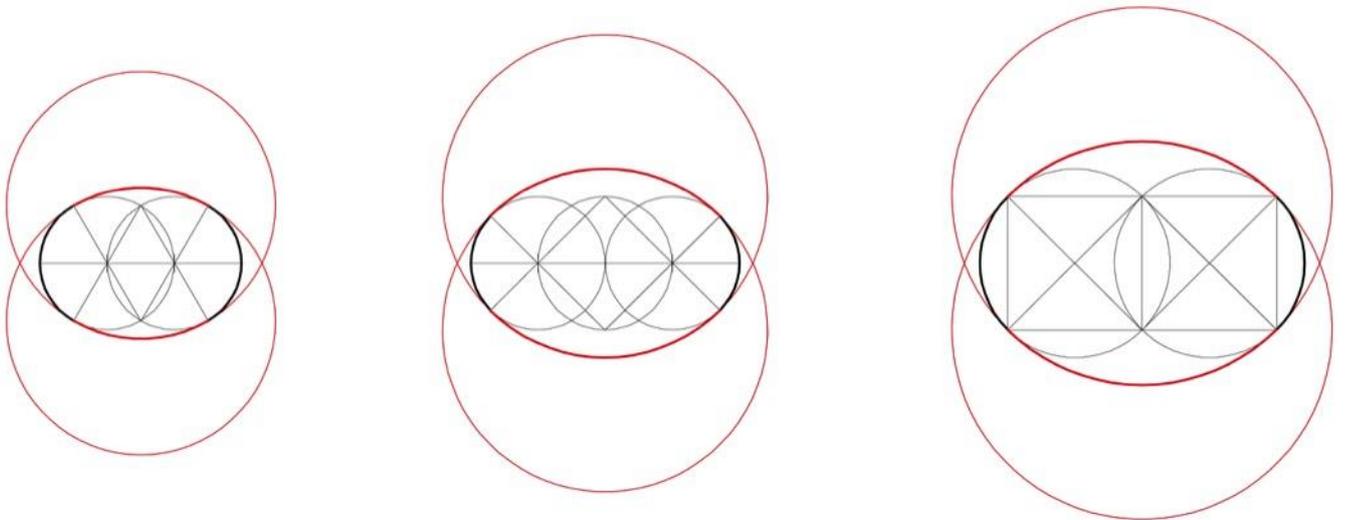


Fig. 4. An oval curve, i.e. a polycentric curve obtained by the junction of arcs of different circles. Some geometric constructions of the oval with ruler and compass. The first two starting by the major axis dimension. The arcs of different rays are highlighted with different colours.

The condition on the tangent line at the connection points implies that the polycentric curves are differentiable.

The property that is exploited in the design of the colonnade of St Peter’s square involves the normal lines to the curves. The normal lines to a circle have the direction of the radius, so they all meet at a single point, the center.

On the other hand the geometric locus where the normal to an ellipse meet *is not* a single privileged point. The normal lines do envelope a curve, called the evolved of the ellipse.

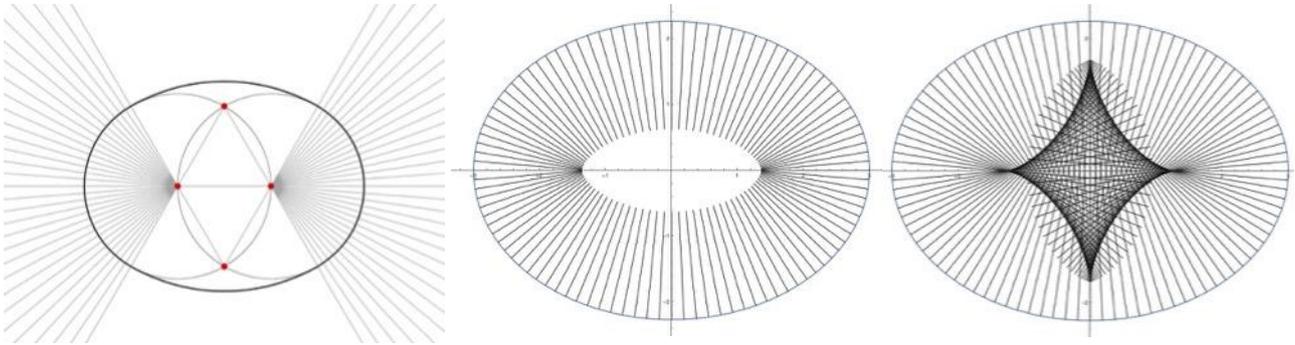


Fig. 5. Left: the oval used for the design of St Peter’s square, and the normal lines from the centers of the smaller circles. Center and right: the normal lines to an ellipse do not meet all at one point. The enveloped curve which can be seen on the right: hand side is called evolute of the ellipse. The center and right pictures where realized with the software Mathematica by Corrado Falcolini

The way this property is exploited in the design of St Peter’s square is shown briefly in picture 6 and will be discussed in sections 4 and 5. The perceptor positioned on the the center of the circular arc of the colonnade (left picture), where all the normal lines meet, sees one column for each row: the columns are aligned along the normal. Right picture: the shape is elliptic and the perceptor on the focus. The normal lines do not meet in one point, so there is not a point from which the visitor could perceive the same effect.

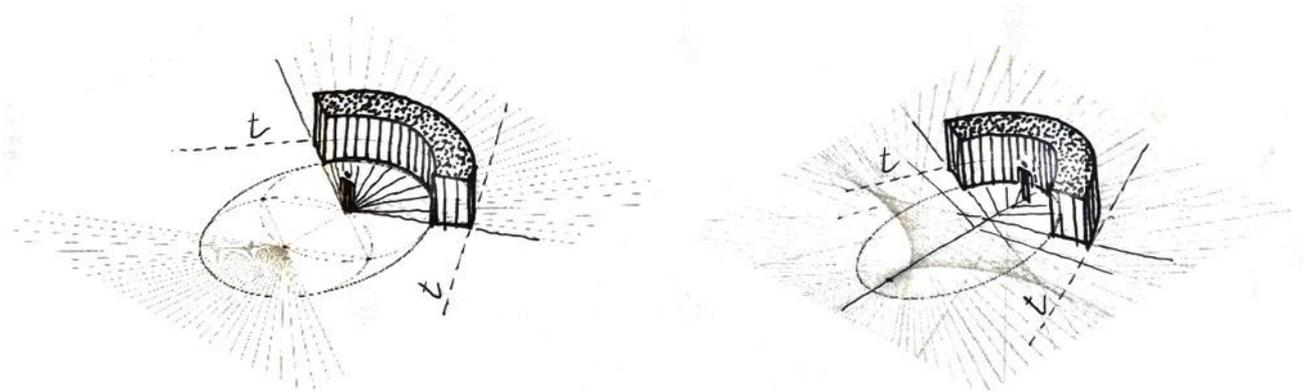


Fig. 6. The sketch shows the perceptual effect of the visitor looking at the colonnade from the center of the circular arc of the colonnade (left), drawing the plan as the actual ovate. Supposing that the plan was elliptical, the visitor being on the focus (right)

3 Ellipse and ovate in architectural culture

There is an ambiguity of use that often occurs between the terms ellipse and oval. In the historic architectural literature related to the manuals and treatises, the expression “elliptic” is used to address a curved geometry, not circular, in other words any unspecified oval.

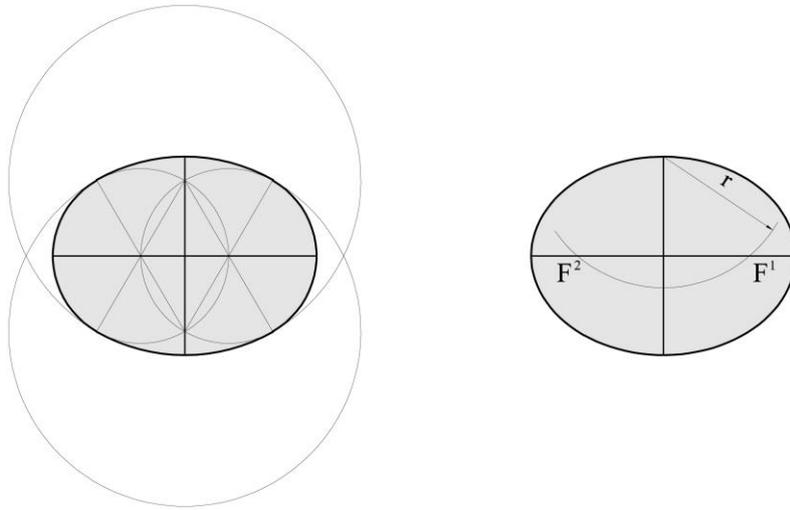


Fig. 7. Left: An oval and an ellipse with the same horizontal and vertical dimensions. The segment r has the length of half of the major axis

Migliari [20], in his studies on curved geometries taken from existing historical architecture and construction practices, points out the spread of this ambiguity; he shows examples from treatises, starting from the ones of the Renaissance, such as that of Peter Cataneo 1567 [7 Libro VII, proposizione XIV].

The two curves are similar in their aspect but their tracking is very different and so are the spatial implications of this on the perceptual level. If one traces an ellipse and a round ovate with the same dimensions (length, width) the difference between the two curves appears minimal (fig. 2).

The reasons for the choice of an oval with respect to an ellipse are, in an architectural and spatial framework, therefore of other kind: ease of tracking, convenience in the execution on the construction yard, perceptual and spatial factors.

In any case, the differences between ellipses and ovals for the usual proportions are so small [...]. Even the most precise mensuration does not serve to settle the matter. It is not a matter of mensuration, but of the history of building traditions. [17]

Like many authors uphold [17, 20, 22], the reasons for such a widespread use of polycentric curves in architecture are related to practical problems, as demonstrated by their spread in the Renaissance treatises (from Serlio onwards).

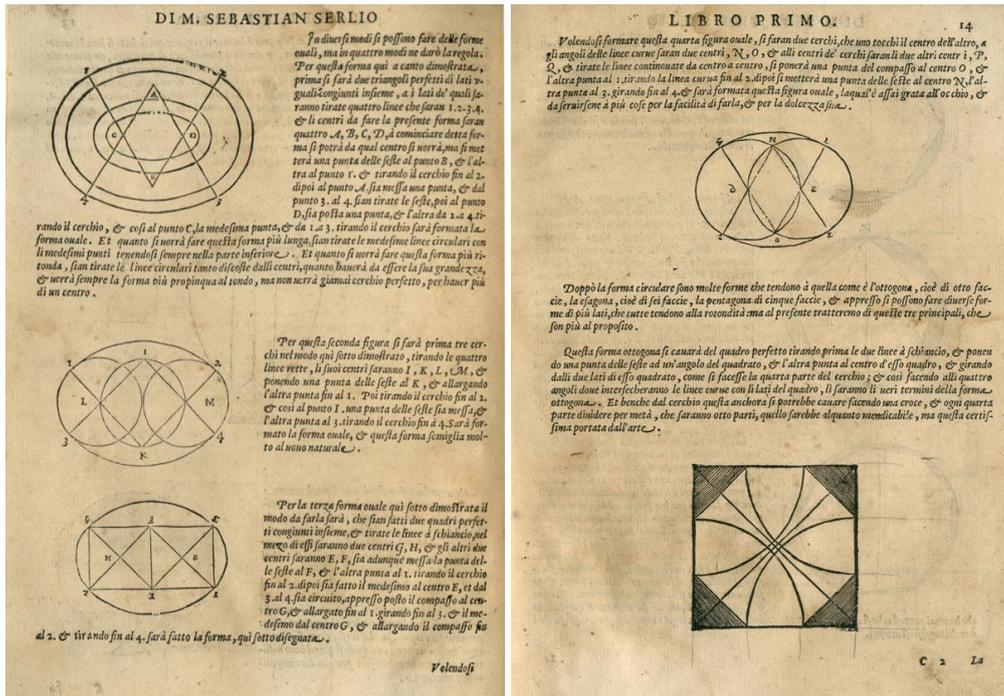


Fig. 8. The four geometric constructions of the oval in the Treatise on Architecture by Sebastiano Serlio (1584) [30]. The Treatise, like other analogue texts, was an asset of operational and stylistic knowledge for the architects of the next generation

The mathematicians Mancini Proia and Menghini discuss in [18,19] the introduction and the use of the oval and elliptic shapes in baroque architecture, and the meaning which was attributed to these two terms. The great architects of Baroque were influenced by the astronomic new theories of Copernicus (1473-1543) and Kepler (1571-1630), and “they become infatuated of the elliptical shape“ (cit [19] pag 334). It was Sebastiano Serlio, an architect, artist and geometer who writes about the geometric constructions of ovals which will help the architects in their design. From the citation of Serlio in [18] we understand which was the use of ellipses and ovals in architecture (here we report the transposition of the author of [18] of the archaic text of Serlio, first book pag. 11)

“vuole l’architetto fare un ponte o un arco, o anche una volta di minore altezza del mezzo cerchio, cosa che molti muratori hanno certa pratica e col filo fanno grandi volte quali corrispondono all’ellisse e si raccordano con alcune ovali fatte col compasso. Nondimeno se l’architetto vorrà provare teoricamente di essere indirizzato dalla ragione deve tener conto di questa linea.” [18, pag 65]

“the architect wants to make a bridge or arch, or a vault of lesser height of the half circle, which many bricklayers have some practice and with wire they make great vaults which correspond to the ellipse. They then connect them with some ovals made with a compass. Nevertheless, if the architect wants to try theoretically to be addressed by the reason it must take in account this method.” [18, pag 65]

Serlio does not expose any of the properties of the ellipse, neither does he report the “gardener’s method” to track this curve (fig. 3). He describes a method to draw an ellipse by points, and

observes that in any case the resulting points should be connected drawing by hand, which is not simple. Therefore, he points out that "in a number of ways you can track an oval, but I will expose only four ways".

Oval shapes were developed in order to approximate the ellipse with portions of circles, so they were born from the very beginning to solve practical issues. Specialized texts included geometric constructions for the tracing by points and instruments for the continuous tracking of ellipses, but these methods were not suitable for drawing at large scale and carried theoretical and practical issues such as measuring portions of an ellipse, or making a large number of different bricks to cover an elliptical arch [20, 22].

We observe also that it was necessary to wait until the nineteenth century to find a veritable repertoire of curves inside a treatise on architecture (see for example Rondelet [31] section devoted to stereotomy).

The debate between oval or elliptical covered many popular historic buildings: the Roman amphitheaters, (see the debate on the forum Nexus journal [40] and [22, 23,10]); some churches, from the late Renaissance onwards [14], and Baroque churches such as San Carlino by Borromini, [6,12] or San Andrea al Quirinale of Bernini [24]; Bernini's colonnade in St. Peter's Square [3, 15,16]; the Ellipse in Washington [15].

An ambiguity of use between the terms ellipse and oval remains in some kind of literature, as touristic literature and some school textbooks. For example, the Coliseum and the plan of St Peter's square are both described as elliptical shaped in [36,37,38,39]. Moreover, in touristic guides (in the four we consulted) St Peter's square is described as elliptical [32,33,34,35].

We found a further ambiguity in the bestseller book "Angels and Demons" by Dan Brown [5].

«Two fountains flanked the obelisk in perfect symmetry. Art historians knew the fountains marked the exact geometric focal points of Bernini's elliptical piazza, but it was an architectural oddity Langdon had never really considered until today. It seemed Rome was suddenly filled with ellipses, pyramids, and startling geometry»

[5 pag 73]

4 The geometry of St Peter's square in Rome.

As it can be seen from an hand-drawn sketch for St. Peter's Square design, Bernini adopts very soon a flattened shape curve.

Fig. 8 (right) reproduces the drawing of the square realized by Fontana in 1694, a few years after the end of the works for the building of the square. In the notes accompanying the figure (top left), still we find the habit to use either the term ellipse or oval. In the notes, in Section A, "*Elipse quale racchiude li portici*" (ellipse which encloses the arcades) and, Section B, "*Elipse di data proportione quale racchiude l'anfiteatro Flavio cioè Colosseo di minor capacità del Vaticano*" (Ellipse of proportion able to enclose the Flavian Amphitheatre, the Coliseum, smaller than the Vatican capacity).

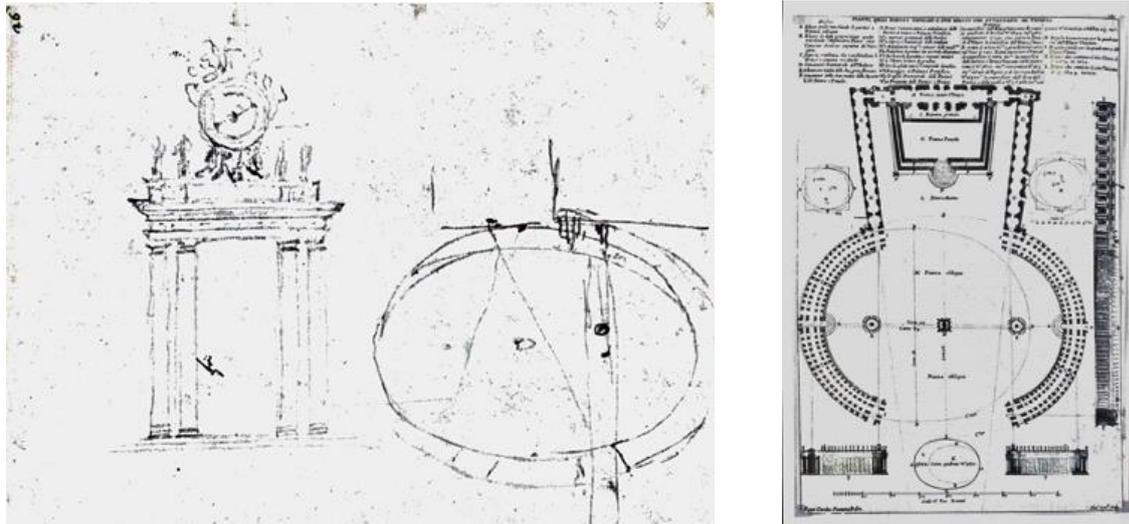


Fig. 9. Left: hand drawing by Bernini (1957, Roma, Bibl. Vaticana, cod. Chigi a I 19, f. 26r) [2,3].
 Right: engraving, Carlo Fontana, 1694 [11]

The author in [18] points out that “Bernini to substantiate the elliptical shape, poses the two fountains where more or less should be the foci of the ellipse which has the same axes of the ovate”. So in literature we have found two possible geometric interpretations on the design of the square:

A_ the oval geometry of the perimeter of the colonnade, with the smaller circumferences centers positioned where the existing porphyry discs are;

B_ the perimeter of the colonnade suggests an ellipse, with the two foci close to the so-called "twin fountains".

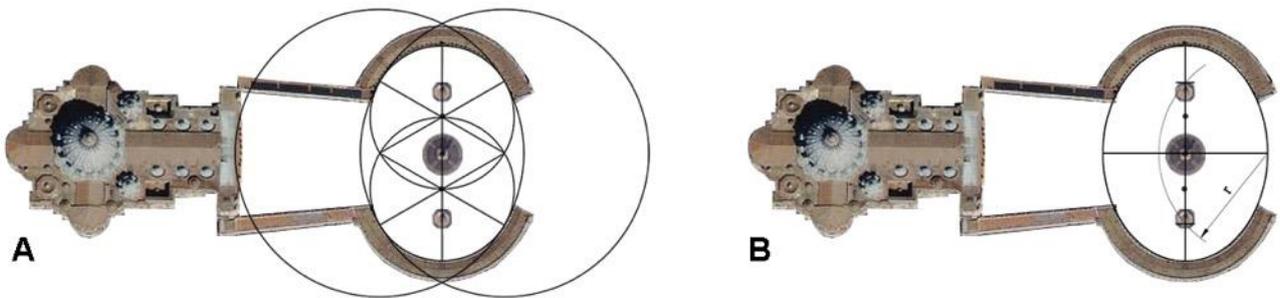


Fig. 10. Reconstructions of geometric assumptions. (A) the perimeter of the oval colonnade, the smaller circumferences centers in correspondence of the existing discs of porphyry. (B) the hypothesis of elliptical colonnade, with foci close to the so-called "twin fountains"

In the historical-critical type literature we found no doubts about the interpretation of the oval design of the square. The fundamental studies of the Roman Baroque, from those of Wittkower [25], to Fagiolo dall’Arco [11], to Brandi [4], speak of ovate and other studies [3,11,16] identify accurately the polycentric construction adopted by Bernini.

5 The relation between the geometry of the plan and the spatial development.

The specific properties of the oval were well known and were deeply exploited in the design of St Peter's colonnade.

From the point of view of the perception of the form [1,4,11] polycentric constructions allow to preserve the typical spatial tensions of the ellipses, but use circular curves which are more practical to treat on the architectural plan. A geometric analysis of the shape of the square can be done through observation on the premises. The perceptual experience of the spatial phenomenon makes explicit the oval geometry of the building.

In St Peter's square the theatricality of the Baroque city can be already appreciated before being swallowed up from the breadth of the square. Actually, when coming from the Borgo district, the colonnade looks dense and almost impenetrable, a forest of backlit columns.

Coming from *Via di Porta Angelica*, and not from *Via della Conciliazione*, through the arches of the *Passetto di Borgo*, it is impossible to imagine the void that opens a little further. Almost as walking in a forest, the visitor passes the shafts of the columns, one after the other. Then he passes through the thickness of the porch; at the end of it he almost falls into the vacuum of the square.

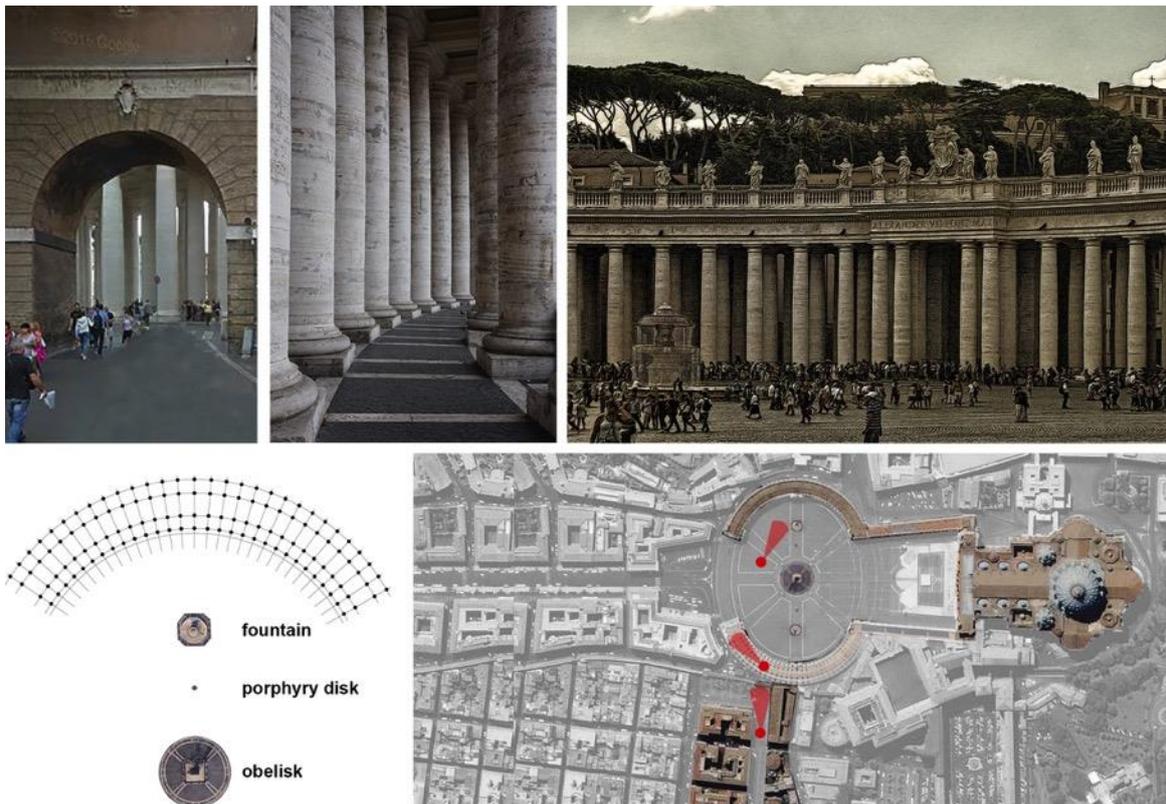


Fig. 11. How to access and how to get around in St. Peter's Square to enjoy the spatial properties coming from his plane geometry. From left: the arrival from Borgo district; the crossing of galleries of the colonnade; the arrival in the square

Crossing the square, that forest of ever changing columns, which are apparently disordered, becomes transparent and ordered only from two observation points, placed on opposite positions

with respect to the obelisk in the center of the square. These two notable points are now marked on the ground by two porphyry discs: on the edge of the disc, the writing “center of the colonnade”.



Fig. 12. The porphyry disk and one of the sundial nodes which mark the pavement of the square

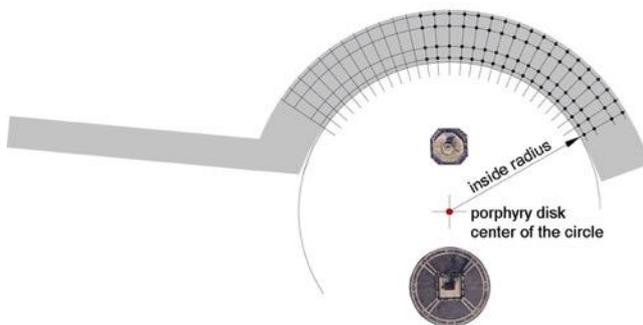


Fig. 13. Image taken by the observer positioned on the porphyry disk.
The development of the plane geometry in the physical space.

On the porphyry disk, the columns are aligned along the normal to the circumference and appear as if they were one. When you start moving into the square, even slightly, the columns in the rows appear again

The design of the colonnade is dictated by the circle geometry: joining any point of a circle with the center, you get the radius, that is a straight line perpendicular to the curve. The columns are aligned along the rays and arranged on four arcs of concentric arcs of circumferences. For this reason, the observer positioned in the center of these circles, marked on the ground by the marble disk, sees only the columns on the innermost curve. Here the multiplicity of three galleries vanishes, the four rows of columns become one. So, the phenomenon that is appreciated in two specific points of the square, where the four rows of columns appear aligned in a single row, allows to acknowledge in the three dimensional space the underlying geometric consistency.

On the other hand, as soon as we rotate our gaze to the opposite side of the square, this unifying view vanishes and again we see a forest of shafts. As soon as we step outside of the porphyry disk, the single rows are broken, the columns become immediately visible as a forest. The disk has a ray of not more than 30 cm. The ray of the circle that defines the colonnade is about 80 meters. A step outside of this small disk means that the perceptor finds himself positioned not more than 50 cm from the geometrical center of the semi-circle, a very small distance if compared to the radius. The perceptive effect is very remarkable.

If the curve was an ellipse, the same perceptual effect could not be achieved, since the perpendicular lines to an ellipse, do not converge all at a single point (see fig. 5-6).

6 The geometric flash mob



Fig. 14. The geometric flash mob in St. Peter's Square

In the occasion for “European Researchers’ Night 2015: exploring science, having fun” we had a geometric flash mob in St Peter’s square. The activity was inspired by a “Progetto Lauree Scientifiche”, a project funded by the Italian Ministry of Education, which dealt with mathematical drawing machines for tracking conic curves, realized by the two authors together with Laura Farroni (Architect, Researcher in the Department of Architecture, Roma Tre University).

Actually, the first flash mob was experimented by the first author and her students. Since it was a success, it was also proposed for the European Researcher's Night together with www.formulas.it, a group of mathematicians and architects from the Department of Architecture of Roma Tre University.

The flash mob was preceded by a "Scientific trip" (in the picture, the two authors speaking about the geometry of the square, in particular about ovals and ellipses).



Fig. 15. "Scientific trip". In the picture, the two authors speaking about the geometry of the square

Before entering the square the visitor do not perceive the enveloping geometry that characterizes it. For this reason, the flash mob started from the neighborhood: so, the surprise effect is more conspicuous and consistent with the theatricality of the Baroque space. Once through the colonnade, the group was guided towards the obelisk and invited to look around, to let them develop their own perceptive relationship with the square and with the spatial scales involved. The colonnade appears at this moment still very consistent, with rows of columns that alternate between the opening and closing of the intercolumns, which are still deep and in the shadow. At the same time, at the top, the colonnade profile, linear and continuous, draws the curved geometry of the plant.

Only after this perceptual experience, gradually, the group was invited to move forward and to conquer the center of the circle marked on the ground by the porphyry disk. Now the relationship between the planar geometry and the physical space is clear and surprising. What seemed obvious on the plan, becomes plastically almost magic. This result that can only be appreciated in this way, walking on foot, moving the eyes, trying to look for a physical contact with the object, reclaiming the public space.

The group of participants was then asked to guess how much could the square be wide. The given answers were quite much under the real measure. Then the group was proposed to measure one of the rays of the "ovato tondo" by forming a line of people, actually to experiment an anthropomorphic sizing. The group was not enough numerous to cover the ray, so other tourists joined the line. It took 58 people, standing with their arms wide. The average arm width is 1.50 mt, so the estimate of the ray was 87 mt. The geometrical composition of the ovato shows that the width of the whole square is three times the ray, so 261 mt. The oval is actually 240 meters wide [11, plate 166], so the anthropomorphic measurement of the amplitude reached a startling precision. Knowing the geometry allows to involve fewer tourists to measure the square.

A final remark: during the flash mob the police officers in the square asked us what we were doing. We explained to them that we were from the School of Architecture, and we were doing a scientific experiment. They let us do and were amused by our intention of forming a human line. We don't actually know if nowadays the flash mob could be done again, thinking of international facts that sometimes lead to stiffen the security measures. We hope that it will be possible to perform it again. But the view of the colonnade will always be possible and the wonder of its geometry always be appreciated.

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