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IMPERIAL ROMAN COCHLEAR COLUMNS AND THEIR HISTORICAL NARRATIVE FRIEZES

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Abstract. Cochlear columns in imperial Rome bore the historic visual narration of campaigns, battles, victories and conquers by emperors. This was possible as a long sculpture narration evolved along a helical frieze, permitting representation without aberration or visual distortion, due to the locally Euclidean property of the surface. We also describe the narrative correspondences relating to the vertical periodicity of the helix.

Keywords: locally Euclidean geometries, cochlear columns, helices, Roman art

Mathematics Subject Classification: Primary 00A06, 00A66, Secondary 00A67

1 Introduction

We present here a study of the surface of cochlear columns in imperial Rome. Helices appear to be part of what we have called *persistence of forms*, a repertory of shapes handed down over centuries and geographies [5]. In [7] we approached the three-dimensional study of medieval tortile columns, which are quite interesting for us, because they are complicated, being the compounding of several different three-dimensional curves. Moreover, they are quite torqued, in a mathematical sense. This compound aspect commanded a subtle work of 3d survey methods, and in particular we used point cloud documentation to infer the horizontal sections.

In the present paper we discuss, instead, a seemingly elementary surface: that of a cylinder spanned by a helix of the same radius. Today we regard the surface of a cylinder as a locally Euclidean geometry; in particular, notwithstanding its obviously curved aspect, it allows to report on it flat representations without any aberration. This attribute was exploited in long historic narrations in imperial Rome, who devised for this the cochlear column. The composition is in fact typically Roman. Mathematically, all aspects of a helix on a cylinder are exploited in the historiated frieze. The frieze develops as a time-line narration of events; the rhythm of repetition of some key characters introduces a diachronicity, being modulated by the vertical rhythm of the helix; we also study the extent the possibility of visual representation and perception without aberration, computing the radius of the neighborhoods where the geometry is locally Euclidean, and intersecting it with that of the visual cone of an onlooker standing in some specific place.

This paper is organized as follows. In Section 2 you will find the history of this celebrative monument, its origins, and its later uses. In Section 3 we discuss the mathematical part: a modern view, as locally Euclidean surface highlights possibilities of visual representation; we also report on the origin of the term cochlear column, which has definitely a scientific derivation. In Section 4, finally, we draw together the mathematical periodicities with a reading strategy for the actual onlooker.

2 Roman Cochlear columns: their historical derivation and fortune, an excursus.

The decorated cochlear column is a celebrative architecture of imperial Rome, and in fact, only developed here, as far as we know. From a figurative standpoint, the historiated column transposes and accommodates the "triumphal painting", a typical Roman tradition, and so figurative language becomes an effective mean of Imperial propaganda. It combines two main typologies of the celebrative art in Roman tradition: the honorary column, and the relief sculpture on a continuous freeze [12].

Fig. 1. Two typologies of the celebrative art in Roman tradition
The relief sculpture on a continuous freeze: A) Parthenon, frieze with Panathenaic Procession, Athens; B) Ara Pacis: processional frieze showing members of the Imperial household, Rome.
The honorary column: C) Column of Phocas, Roman Forum, Rome

Continuous sculpted friezes, on "flat" background surfaces, were also already in use in Greece, e.g. the Panathenaic Procession described on the Ionic frieze of the Parthenon (Athens, 5th century BC)[1]. The large frieze, though, is not so much narrative, as descriptive of a spatially long ritual

[1] This beautiful frieze is so long that, alas, the original has been separated into pieces and scattered around the world: The appearance of entire frieze has been reconstructed in cast in the Acropolis Museum (Athens, architect B. Tschumi, 2008). From its parts held at the British Museum in London, those still onsite at Athens, and several fragments from other Museums and institutions.

procession and games. A Roman example of historic narrative frieze is the relief of the Augustus's procession on the Ara Pacis. The Ara Pacis celebrates the *pax augusti* (peace) "donated" by the emperor to Hispania and Gaul (Rome, 9 BC).

In cochlear columns, object of the present study, the continuous frieze is arranged on a ribbon, winding up the column's shaft; this allows for long narrations of events taking place over a long time span.

The idea of a "figurative ribbon" winding up and spiraling around the column's shaft seems to be new and peculiar to Roman art. Several hypothesis have been set forward to its genesis. We report those cited by Settis [12]:

1. The ribbon is inspired by the *rotulo*, early form for a book, or *volumen* (volume) used in the ancient libraries and consisting of a strip of flexible material so that it could be rolled in various ways. It could be considerably long [1].
2. The ribbon is inspired to the habit of winding some fabric around columns and piers on festivities; the fabric was an ornament, and in turn, could be decorated with images.

We report these two because they are also both congruent with our studies. In fact, the first instance ever of such art, appeared in the Trajan column, situated in the inner courtyard of libraries, this the visual art would reflect visually the *rotuli* contained in the library in their narrative development. On the other hand, a ribbon of fabric can be wound around a cylinder without folds and creases, depending on the relation between its height and the slope of the helix.

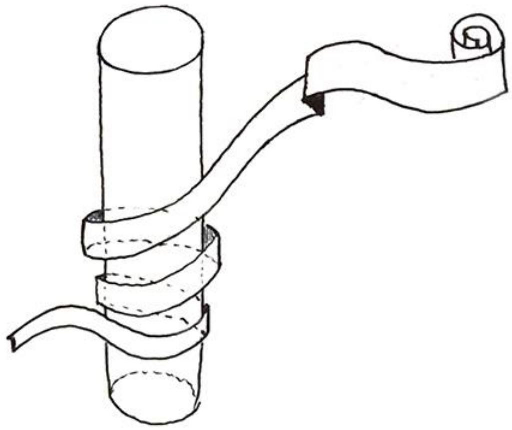

Fig. 2. A historiated frieze on a flat ribbon. The ribbon arranged on the cylinder without loosing its local flatness

The first cochlear historiated column is that of Trajan, built to celebrate the emperor's victory over the Dacians (Rome, Trajan's Forum, 110-113 A.D.). The architect of the forum was probably Apollodorus from Damascus, who also conceived the column. The sculpture are due to one or more sculptors under the guide of a *maestro*, whose name we do not know [2].

The Trajan column was hereafter taken as reference model for other imperial columns:
- In Rome, not much later, was built another historiated column, the Colonna Antonina (180-193 A.D.) or "of Marco Aurelio".

- A couple of centuries later, in Constantinople two more columns were built: the Columns of Theodosius (386 A.D.) and that of Arcadius. Neither one is preserved to our days, but we have some fragments and some historical drawings.


Fig. 3. Trajan's Column, the first cochlear historiated columns (Rome, 110-113 AD) and Column of Marcus Aurelius (Rome, 180-193 AD)

The celebrative cochlear column becomes hereafter a reference model crossing different centuries, styles and narrated historical or political events. Some other examples of celebrative cochlear columns are (Fig. 4):

- Hildesheim, Germany, 1022: the Column of Saint Bernnward, depicting scenes of Christ's life;
- Vienna, Austria, 1715-1737: the two columns flanking the Karlskirche, depicting scenes of the life of Saint Carlo Borromeo;
- Paris, France, 1805-1811: the famous column in Place Vendôme, commemorating Napoleon's victories at Austerlitz;
- Astoria, Oregon Usa 1926, depicting the American epic.




Fig. 4. The mentioned cochlear columns respectively from: Hildesheim; in Vienna; Paris; Oregon

3 Helices, some mathematical aspects.

A helix is the composition of a rigid uniform rotation in a horizontal plane and a uniform translation along an orthogonal axis. Thus defined, a cylindrical helix evolves around a solid cylinder, maintaining a constant slope:

$$X(t) = r\cos(t)$$
$$Y(t) = r\sin(t) \tag{1}$$
$$Z(t) = p\,t$$

Where r is the radius of the cylinder and p is the *slope* of the helix.

We will discuss only the surface of such cylinder, which has zero Gaussian curvature, and therefore we prefer to regard it as a locally Euclidean geometry [10]. A modern way to read a locally Euclidean 2d geometry is to see it as the quotient space of the plane R^2 with respect to an equivalence relation introduced by an isometry, in this case a translation (Fig. 5).

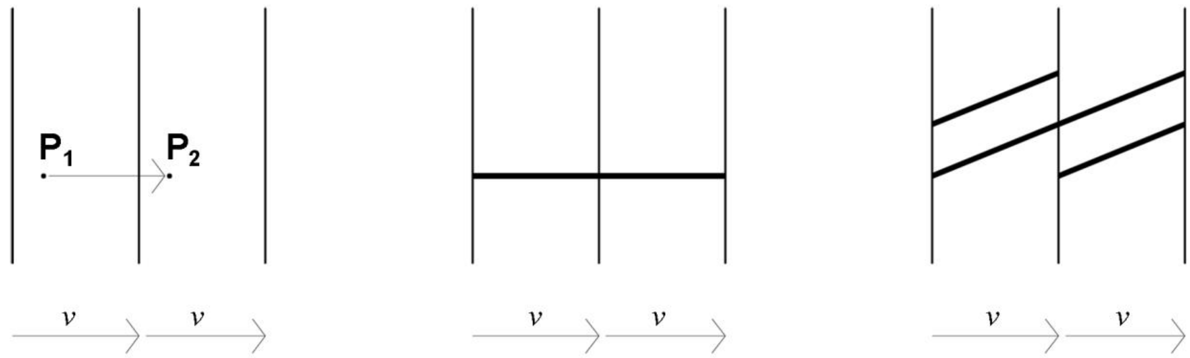

Fig. 5. Surface of cylinder defined as equivalence on the plane: points are equivalent if taken into each other by a translation of vector ***v***

Several sets can represent the quotient space in the plane; when the set is connected it is called the "fundamental domain" of the geometry defined by the equivalence relation. One can then think of the cylinder as the realization in 3d of this fundamental domain, for instance by imagining that we could glue lines that are equivalent under the translation. In such description lines orthogonal to the translation vector, remain as such; lines parallel to the translation vector become finite segments (or circles on the surface). All other straight lines become helices on the final imagined surface (Figs. 5, 6); the two representations (helix on a surface, straight line on a plane) share the same slope. This representation highlights the uniformity of the cylindrical surface, while thinking of a three-dimensional object naturally divides the surface into front and back (not visible), and into more areas depending on perception. Images can locally be represented exactly as on a flat surface, and will be perceived correctly by an onlooker moving around the solid.

To the spectator today, standing at street level, or to our readers, this might seem as a sophisticated geometrical condition, and out of the point: to appreciate the narration, today, we go to printed photographs. On the contrary, we think that a locally Euclidean geometry is implicitly weaved in the construction of the narration. Documentation is available [3, 4] as to the fact that spectators looked at the Trajan column by going around the column on balconies of the two-story Libraries that surrounded it (Fig. 8).

Some go further by thinking that spectators would also climb to next level of the library, but this idea is not necessary to our argument. What is important is to reconstruct what a spectator could see from that exact distance, even without moving, which we will discuss in next section.

There is evidence that besides the linear organization along the helix, that narration was also planned to have episodes read on the same side, in vertical, thus respecting both the parametric order of the helix, and the behavior displayed vertically due to the periodicity of the helix. In system (1) points corresponding to parameter values of $t_1 = t_0 + 2\pi$ are on the same vertical line.

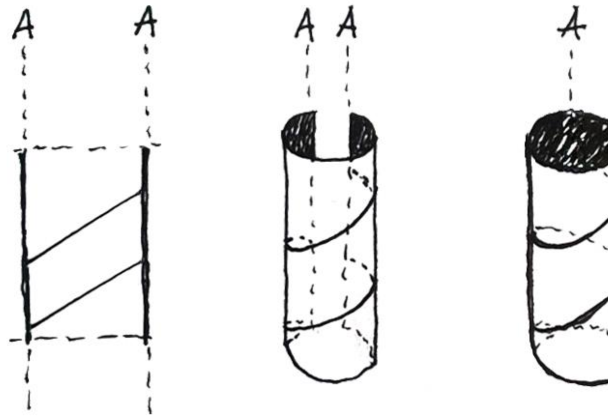

Fig. 6. Recomposition of three dimensional object by identification at the borders

3.1 The "*cochlea*".

A cochlea can denote several objects, all deriving their name from the Greek *koklias*, or the Latin *cochlea*. It was one of the first historical idraulic machines, the Archimedes' screw, devised to lift water and still in use to lift incoherent materials. It has also been used to denote the spiral stair. Mathematically, all the material realizations of *cochleae* have in common the constant inclination with respect to the horizontal plane, which while being functional in the "machine" makes the device geometrically very clear. In fact, representations of drawing machines to plan a crew can already be found in the drawings by Heron of Alexandria, passed on to us via the Persian manuscripts [8]. The early supporter of Galileo Guidobaldo Del Monte, himself a mathematician and astronomer, devoted a whole book to the cochlea [6].

Cochlear columns are called this way both for the helix resulting on their surface, and because internally they carry a spiral staircase (Fig. 7). The stair allowed going up to the summit, to the base of Emperor's statue. Common to all such mathematical and mechanical uses, is a curve or a surface of constant vertical slope. As well known, in the mechanical case the constant slope was functional to lifting materials, much as in the spiral stairs it is to walking up. In our case the constant slope is the mathematical expression of this flat ribbon with parallel borders, i.e. very patently a locally Euclidean surface. In fact, also Giacomo Martines, a historian of architecture, sees in fact the Trajan's columns as an "exercise in Hellenistic mechanics" [9].

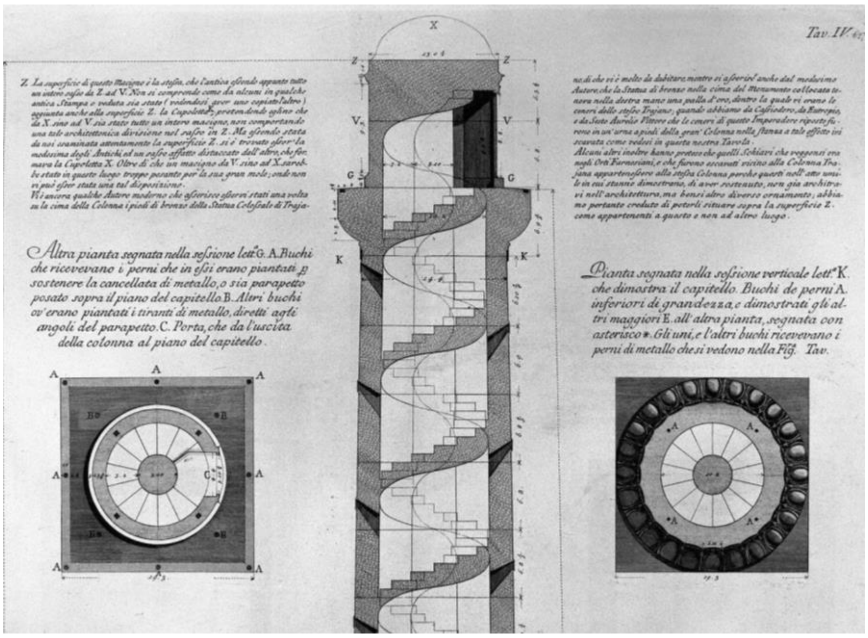

Fig. 7. G.B. Piranesi, Trajan's column (original title: *Trofeo o sia magnifica colonna coclide di marmo [...]*, 1774-79). Vertical section with spiral staircase

4 Mathematical Rhythms, Narration Rhythms.

The present condition of visibiltiy of the column is quite different from the original. Today the onlooker perceives the reliefs either form the level of the present road, or from the level of the ancient Forum. The Column is thus isolated in a wide space, including Piazza Venezia, and can thus be seen from very far away. In ancient times, the Column was in a narrow courtyard (a square of 20mt side), in the trajan's Forum; two libraries were the sides of this courtyard [3, 4]. Fig. 8 gives a reconstruction of this situation. The onlooker had a close-up vision, and some of the reliefs underwent aberrations due to distance, many of which were perceived in foreshortening (Fig. 9).

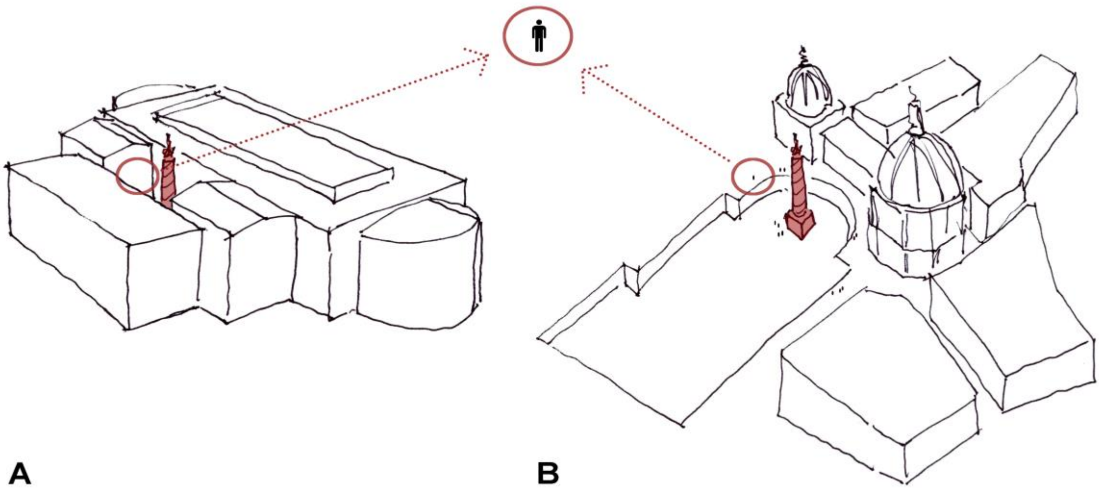

Fig. 8. Onlooker at Trajan's Column:
A) at the windows in the original forum complex. B) at street level in its urban context today.

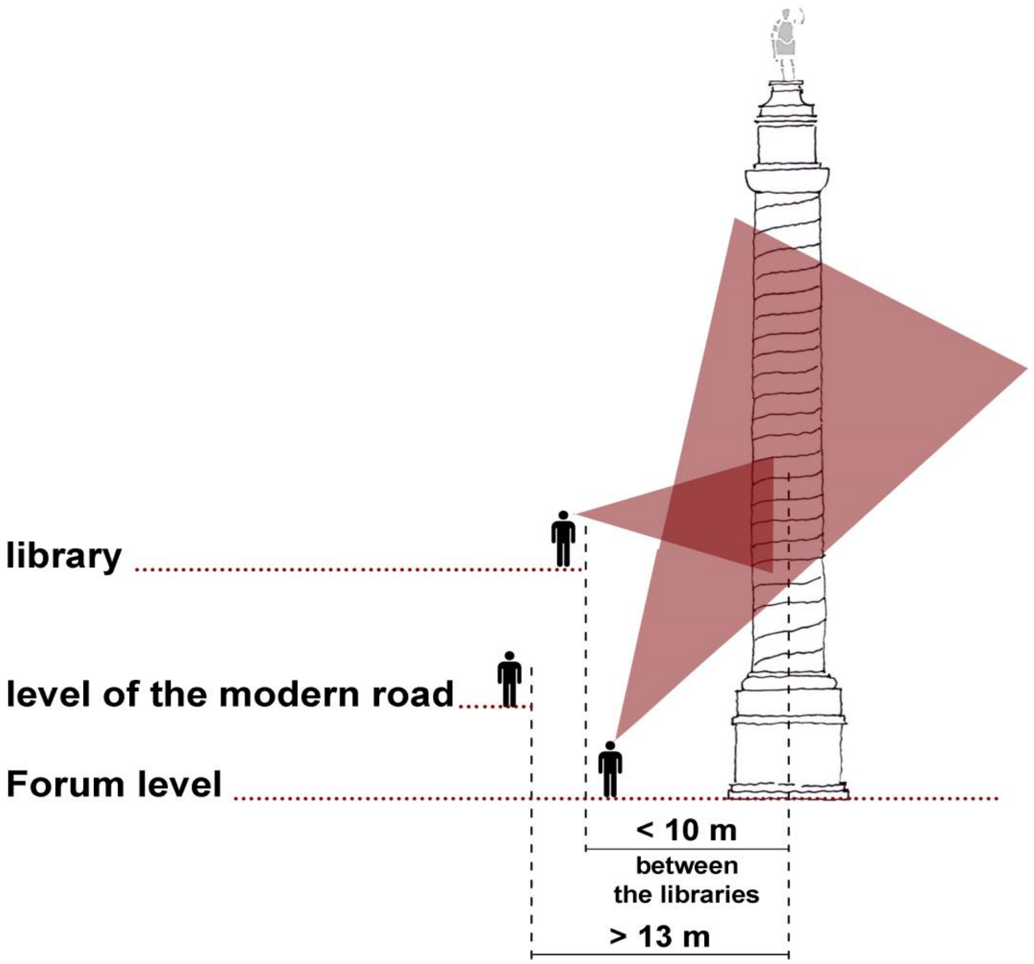

Fig. 9. The two levels at which the ancient onlooker would stand: library level, and Forum level, at the bottom of the Column. The corresponding optical cone is highlighted

A ribbon winding up a column therefore suggest two different paths of reading.

A historical, chronological reading that is continuous and develops along the spiral. The reading follows the narration much as in writing, i.e. from left to right, which, in this case, implies also from bottom to top. One then imagines the onlooker moving around the columns and raising his eyes to follow the narration. His interpretation will be temporal.

A discontinuous order, going from one coil to next, along the vertical. The reading then follows figurative correspondences among successive coils, and gives another interpretation of the events: an a-temporal interpretation, digging into symbols and yielding to propaganda [11].

We will only comment on three examples, but there are more, depending on the side of the column.

The Emperor's image is repeated over and over, in several instances. Some of the scenes are aligned along vertical correspondences. This creates redundancy and a ″visual vocabulary″ [11].

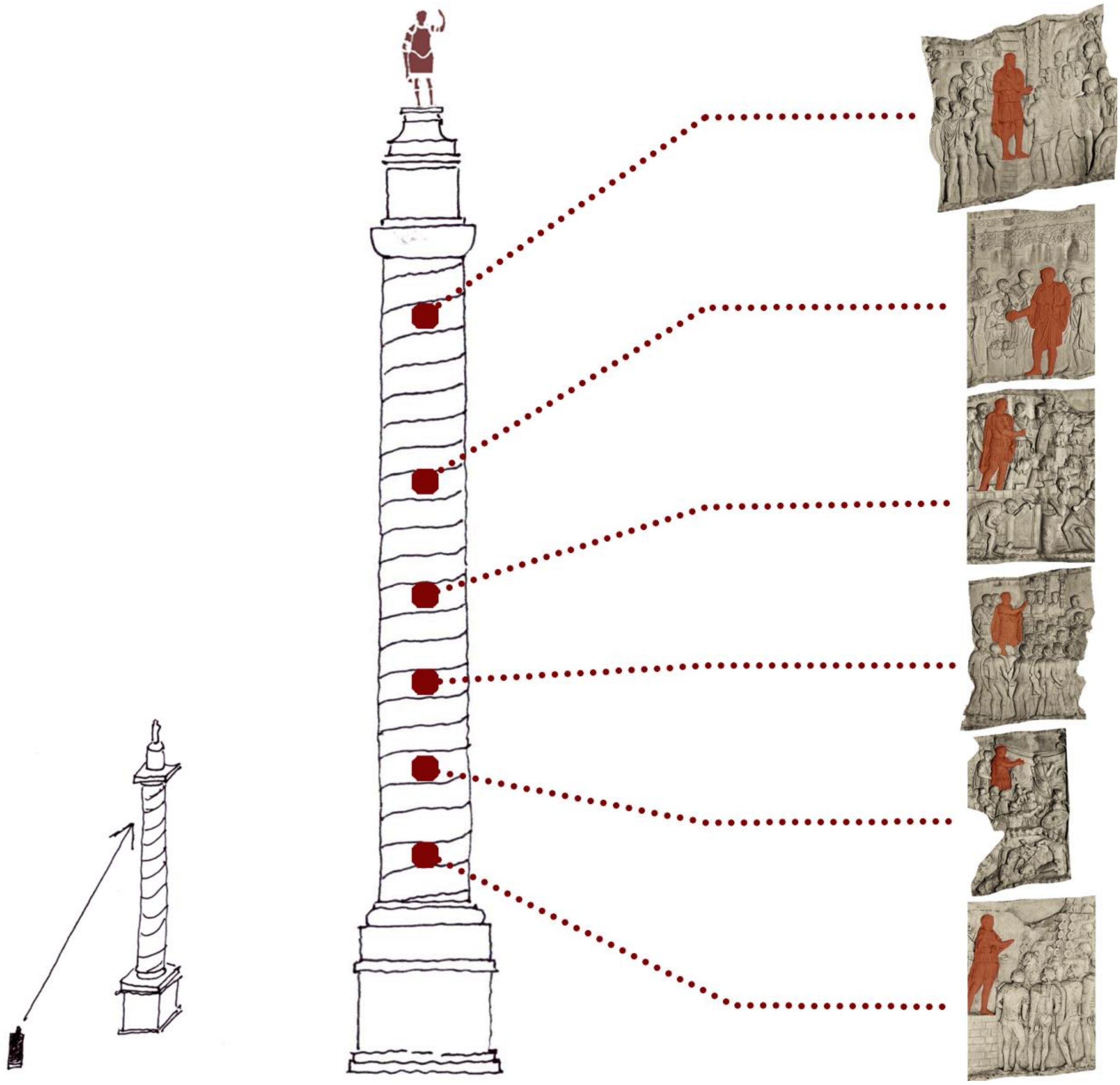

Fig. 10. The Emperor’s portraits, aligned in one strip visually available to a standing onlooker

Celebration of Victory. An a-temporal vertical narration depicts the symbolic unraveling and disentangling of victorious battles. All on the same vertical line, we find, starting from the bottom: the premonition of victory, the Winged Victory, and the suicide of the Dacian king, sanctioning the defeat of the Dacians.

A pontoon bridge is laid for crossing by the troops. The repetition of the scene exalts the sophisticate abilities of military engineering. This type of scene appears only three times in the almost 200mt long frieze; alignment along the vertical favors its understanding.

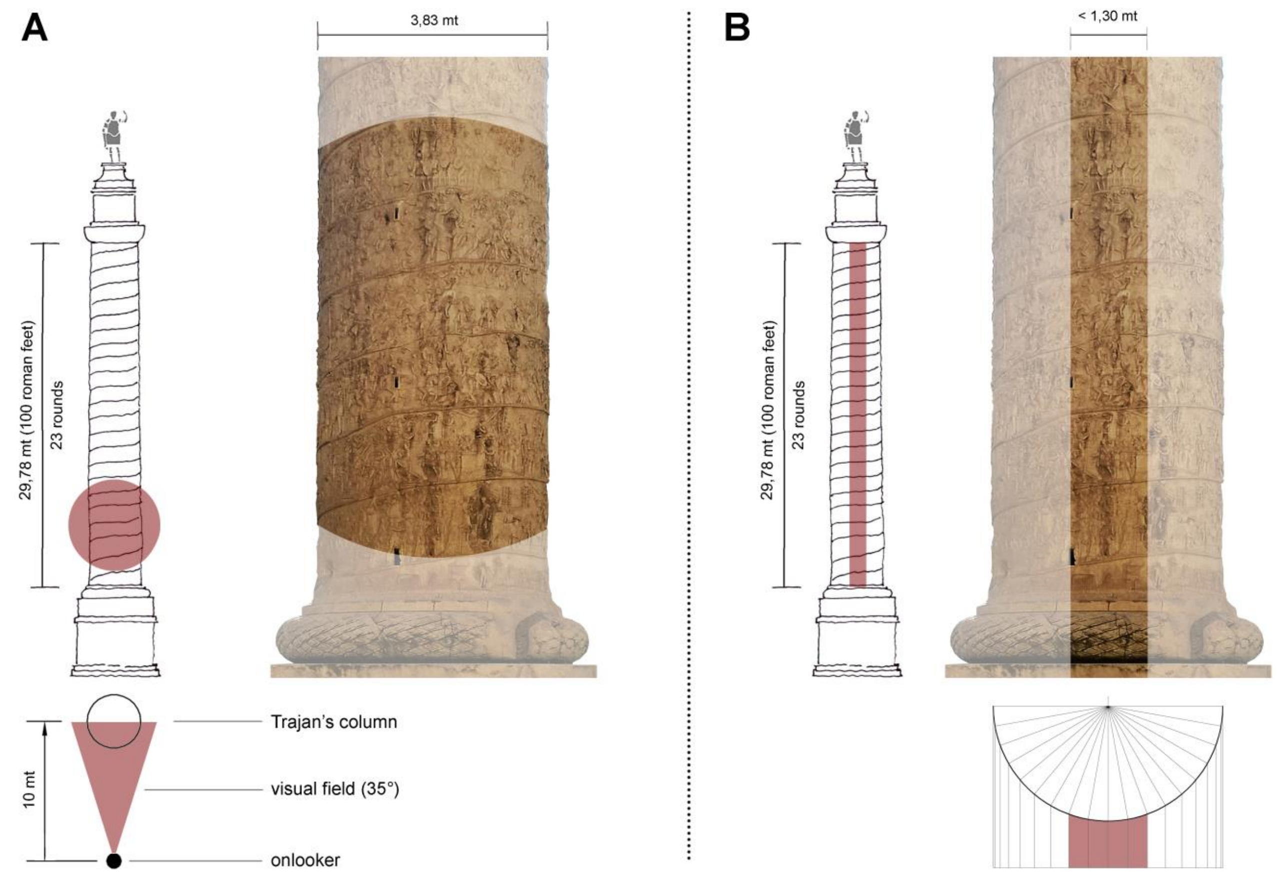

Fig. 11. A) Computed ″visual cone″ of a perceiver standing in original spot.
B) Frontal area: vision not affected by curvature of the cylinder

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