

# PARAMETRIC PLANAR SECTIONS OF A POINT CLOUD WITH APPLICATIONS IN CULTURAL HERITAGE 

FALCOLINI Corrado (IT)


#### Abstract

Given a discrete ("Point Cloud") representation of a surveyed object it is possible to analyse its three dimensional features with the help of several planar sections. We survey a few applications of original algorithms to specific cases of interest in archaeology or architecture such as reconstruction of fragmented objects or symmetry detection.


Keywords: Point Clouds, Planar Sections, Symmetry Detection, Object Reconstruction
Mathematics Subject Classification: Primary 65K05, 53A04; Secondary 97N50

## 1 Introduction

Data sets coming from modern surveys (see [2], [4]) are basically made by a long list of coordinates of points detected on the surface of the object under study.

Each point can be associated to some local feature: color (photogrammetric survey), reflectance (an index of material reflection from laser scanner survey), orientation (average normal to the closest faces of the triangulated surface through the points) but point coordinates are often enough to evaluate its 3D geometrical model and feature.

The algorithms mentioned in this paper have as input the point cloud of a given fragment, or portion of a monument, and as output the best possible sectional curve on a given plane: with the help of several planar sections of an object it is then possible to analyse its three dimensional features.

Applications in Cultural Heritage presented in this paper run from the detection of an optimal fluting column model, in the virtual reconstruction of the Arch of Titus at the Circus Maximus in Rome (see [3], [8], [10]), to the contact probability of two possibly adjacent fragments, in the recomposition and restoration of the fragmented statue of S. Andrea at Stiffe, L'Aquila (see [1], [6], [9]) or the shape and tasselation of Borromini's San Carlino dome (see for example [5], [7], [12]).

## 2 Planar sections of a point cloud

Starting from the list of points over a column fragment, as in Fig.1, and for a given plane we test a segmentation procedure to approximate its curve of section. Starting as in Fig. 1a, we select all those points at a distance smaller than a certain threshold from the fixed plane and we project them (see Fig. 1c) on the plane. Shape thickness in Fig. 1d depends on plane orientation and on the fixed threshold.


Fig. 1. Points on a given plane section of a surveyed column fragment: a) selected points at a distance from the given plane below a threshold; b) selected points in a box; c) projection of selected points on the given plane; d) planar representation of selected points: thickness depends on plane orientation and threshold

Decreasing the threshold we may look for a thinner curve, as regular as possible. Such a curve-like shape will be made of unordered points, difficult to connect but still useful for testing orientation and symmetry.

As a second step we evaluate an interpolating curve with the special feature of being parametric. Among several algorithms we choose to draw a polyline, ordering the selected points, or to apply a continuation method with an appropriate condition.

Another possible issue with a planar parametric curve as output (see [9]) is to find the border of a given set of points $S$ on the plane: this could be found with a convex hull algorithm or with the algorithm shown in Fig. 2.


Fig. 2. a) Properly choosing the maximal circle of the circumscribed sphere connect the points of the set at minimal distance; b) take a uniform set of points on the interpolating parametric curve and iterates the procedure; c) corrections for straight portions of the limiting parametric curve

In order to detect a parametric border of S (small black points in Fig. 2) one can start with a uniform distribution of base points on a circle containing S, select for each base point the element of S at minimal distance (large blue points in Fig. 2a) and construct an interpolating parametric curve.

On this new curve take a uniform distribution of new base points and iterate the procedure: some corrections might be useful to prevent stright pieces of the limiting parametric curve which prevent to move closer to S . This algorithm works also in three dimensions.

## 3 Parametric curve of sections and inflection points

The first example refers to the application of an automatic procedure (see [7]) for the extraction of a piecewise regular parametric curve of section directly from the point cloud.

To detect a moulding profile of a surveyed fragment of the Arch of Titus at the Circus Maximus in Rome (see Fig. 3) we first have to select a relevant plane of section: if the point cloud is properly oriented we could choose a vertical plane which optimize points selection.

The correct orientation of the fragment, or that of the relevant plane of section, can be found analysing the normal components in the standard point cloud data: at any point of the cloud is associated an average normal to the closest faces of the corresponding triangulated surface. For instance it is possible to automatically select planar regions of the surface analyzing tstatistical properties of nearby normal directions and to check precise alignments using specific properties and symmetry.

We then look for a parametric curve fitting the selected points, like in Sec. 2, on the chosen plane (see Fig. 1).


Fig. 3. Left: points on a molding section of a surveyed fragment. Right: molding interpolating curve of a column base section, inflection points detection and final geometrical reconstruction

Special points on the parametric curve, such as point of discontinuity in the direction of normal vectors and inflection points, allow an automatic geometrical moulding construction which can be translated in feature elements of the database and used in matching procedures.

As another example we show in Fig. 4 a superposition of vertical parametric sections to check symmetries in coffers dimensions and placement in Borromini's San Carlino alle Quattro Fontane dome.


Fig. 4. Left: point cloud of a portion of S. Carlino's dome and selected points on a vertical section. Right: vertical sections comparison. The distance between two different curves can be measured using their parametric representation

Two different sectional curves represent the central molding profile of coffers placed in different positions on the dome. Curve profile gives informations on single coffers (octagons and crosses) size and its simmetric properties, but also on their mutual orientation and the geometrical shape of the dome intradox.

The distance between the two curves, and other geometrical features, can be measured using their parametric representation.

Comparison between the curves gives informations on simmetric properties of the dome and indirectly on its construction procedure.

## 4 Optimal fluted column model

From the survey of a ruined column drum (the same shown in Fig. 1), the second algorithm example shows how it is possible to find an optimal fluted column model (see [10]).

A simple n-fluted column section model can be easily constructed: take a circle $C_{0}$ of radius $\mathrm{r}_{0}$ and a smaller circle $C$ of radius $r$ centered on a point of $C_{0}$ and call P one of the two intersection points of $C$ with $C_{0}$ (see Fig. 5). The coordinates of the point P are then immediately computed as a solution of the system in Fig. 5. Also the angular distance $\theta$ between consecutive flutes is easy to compute and these formulas allow a parametric section model of $n$-fluted column of radius $r_{0}$ depending only on the size of $r$.


$$
\left.\begin{array}{l}
\left\{\begin{array}{l}
x^{2}+y^{2}=r_{0}^{2} \\
\left(x-r_{0}\right)^{2}+y^{2}=r^{2}
\end{array}\right. \\
x_{P}=r_{0}-\frac{r^{2}}{2 r_{0}}
\end{array}\right\} \begin{aligned}
& \theta=\frac{2 \pi}{n}-2 \arccos \frac{x_{P}}{r_{0}}
\end{aligned}
$$

Fig. 5. n-fluted column section model: $P$ coordinates and the angle $\theta$ between consecutive flutes are easily computed

The point cloud of the surveyed column fragment of Fig. 1 are shown in Fig. 6a projected on a vertictal plane. The projection is not completely orthogonal to the best cylinder containing the fragment thus the black border of the projection (see Fig. 6a) appears thicker than needed.

In Fig. 6b the same fragment is shown with a better orientation and cutting out the drum basis.


Fig. 6. a) Projected points of the drum on a plane: the projection is not completely orthogonal to the best cylinder containing the fragment; b) The same projection with a better orientation and cutting out drum basis. Radii $r$ and $r_{0}$ can be estimated on a proper scale

Radii $r$ and $r_{0}$ can then be estimated on a proper scale and the section model used to reproduce the original column, taking into account historical and architectural order information about the shape of a planar section containing column axis (entasis, tapering).

The n-fluted column section model of Fig. 5 agrees very well with the projected points of Fig. 6b whose border represents an optimal vertical section of a ruined fallen column fragment. Note that the size of the drum which has been projected is around 1 meter.

Comparing a cylindrical projection of the oriented point cloud with several section models, allows a very precise measure of column and flutings radii and then a possible drum reposition along the virtually reconstructed column.

## 5 Lines of fracture

The third application is for the broken statue reconstruction of S. Andrea at Stiffe, L'Aquila (see [9]) and shows the use of section curves to detect lines of fracture which could be tested for matching probability.

Points on a given planar section of a surveyed statue fragment (see Fig. 7 and Fig. 8) are obtained first as projection on the given plane (as in Sec. 2) and we look for a parametric curve which interpolates these points using an automatic procedure. Then we take uniformly spaced points on the parametric interpolating curve and use automatic inflection points detection to select special points depending on geometrical properties of the parametric representation (see Fig. 7).

These nodal points on many different parallel sections might line up to evidentiate a line of fracture (see Fig. 8) where the normals to the surface of the 3D-model of the surveyed fragment change
suddenly direction: the problem of reconstructing a broken statue from its pieces can then be attacked by testing high matching probability checking corresponding curves and not only surfaces of contact.


Fig. 7. Points on a given planar section of the surveyed statue fragment of Fig. 8: a) projection of selected points on the given plane; b) uniformly spaced points on an interpolating curve close to the selection; c) parametric curve and automatic inflection points detection

All measures have been optimized up to a certain error using the processed data and have been verified and improved after any new fragment addition or change in a suitable database.


Fig. 8. Left: a fragment of the statue of S. Andrea at Stiffe, L'Aquila. Right: parallel planar section curves with some inflection points (the same as in Fig. 7c) lined up on lines of fracture.

Some of the original algorithms mentioned here have been also applied in different contexts.

Parametric planar sections of a point cloud are good starting point to analyze special features of a given surveyed architectural or archaeological object. Some interesting questions about the object at study can be partially answered using algorithms to construct the closest parametric curve to the points based mainly on point cloud coordinates.

## References

[1] Huang Q.X. et al., Reassembling Fractured Objects by Geometric Matching, ACM Trans. Graphics, vol. 25, no. 3, 569-578, 2006.
[2] Dellepiane M. et al., Multiple Uses of 3D Scanning for the Valorization of an Artistic Site: The Case of Luni Terracottas, Proc. Eurographics Italian Charter Conf., Eurographics, 2008.
[3] Thuswaldner B., Flory S., Kalasek R., Hofer M. Digital Anastylosis of the Octagon in Ephesos, Journal on Computing and Cultural Heritage, 2(1), 1-27, 2009.
[4] Scopigno R., Callieri M., Cignoni P., Corsini M., Dellepiane M., Ponchio F., Ranzuglia G, 3D models for cultural heritage: beyond plain visualization. IEEE Computer, 44, 2011.
[5] Falcolini C., Vallicelli M. Modelling the vault of San Carlo alle Quattro Fontane, Aplimat Journal of Applied Mathematics, 4, 143-150, 2011.
[6] Palmas G., Pietroni N., Cignoni P., Scopigno R. A computer-assisted constraint-based system for assembling fragmented objects, In Proc. of Digital Heritage 2013 International Congress, vol. 1, IEEE, pp. 529-536 .
[7] Canciani M., Falcolini C., Saccone M., Spadafora G. From point cloud to Architectural models: algorithms for shape reconstruction. ISPRS Archives, XL-5/W1, 2013.
[8] Canciani, M., Falcolini, C., Buonfiglio, M., Pergola, S., Saccone, M., Mammı, B., Romito, G. Virtual Anastylosis of the Arch of Titus at Circus Maximus in Rome. International Journal of Heritage in the Digital Era, 3(2), 393-412, 2014.
[9] Canciani M., Capriotti G., D'Alessandro L., Falcolini C., Saccone M. The recomposition of fragmented objects: the case study of St. Andrea statue at Stiffe, L'Aquila, Le vie dei Mercanti, XIII Forum Internazionale di Studi, Capri, 2015.
[10] Falcolini C., Talamanca V. Modelli geometrici per nuvole di punti, In Atti del Convegno Mathematica Italia UGM, Napoli 28-29 maggio 2015, ISBN 978-88-96810-04-0.
[11] Falcolini C. Algorithms for Geometrical Models in Borromini's San Carlino alle Quattro Fontane. Handbook of Research on Visual Computing and Emerging Geometrical Design Tools, 26, 642-665, 2016

## Current address

## Corrado Falcolini, Associate Professor

Dipartimento di Architettura, Università Roma Tre
Via Madonna dei Monti, 40, 00184 Roma, Italy
Tel. number: +390657339621, e-mail: falco@mat.uniroma3.it

