



## **THE FLOOR PLAN OF SANT'IVO BY BORROMINI**

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**Abstract.** Borromini's use of symmetry is extensive, also in three dimensions, in architectural structures, plan and details. Scholars, at times point to symbolical meanings weaved into his compositions. We examine the plan of the floor of Sant'Ivo alla Sapienza, in Rome Italy, and analyze its symmetry group. This symmetry group is that of an equilateral triangle, and is also clearly perceivable as such on the lift of the walls. This merely mathematical analysis agrees with historical documents on the use of the compass by Borromini, and disagrees with a spread interpretation of symbolical order, which would require a hexagonal symmetry. This analysis, in turn, yields a further question on the organization of curved surfaces typical of Borromini.

**Key words.** Dihedral symmetry group, invariance under rigid motion, perceptual analysis

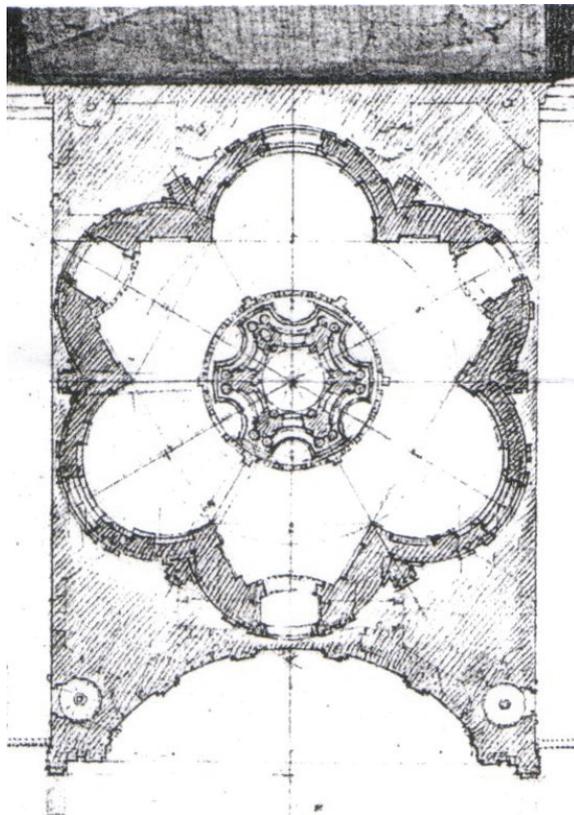
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### **1 Introduction**

Borromini erected the small baroque church of Sant'Ivo alla Sapienza in Rome in a period of time spanning three pontificates, from 1642 to 1660. Over this long period, Borromini and his assistants have made several drawings, and there is a large scholarly literature analyzing their differences [1 and 2, for extensive discussions of the literature]. In particular a wide debate has taken place over the compositional principle on which Borromini has arranged the entire plan of the church interior. In this paper, we are concerned with the geometrical organization of its map floor, and its relevance to perception of symmetry in the entire church interior. We take the standpoint of modern geometry, i.e. the study of invariance under transformations. This is to say we are not concerned for the moment with the historical otherwise very relevant problems, but we analyze the plan through its symmetry group, and study whether and how this invariance is actually perceived by a person standing today in the actual three dimensional space of the church's interior. We think this is a methodological contribution that gives way to a number of different questions. Moreover it is a method that allows an organic discussion of the spectator's perception.

The plan of Sant'Ivo, as it appears (fig.1) in the archives of Albertina in Vienna could well be graphically analyzed, both as an equilateral triangle with some niches, or as a six-pointed star

with some sides curved. The second organization, though, implies precise strong symbolical and cultural references, which many historians see as quite debatable in connection with Borromini. The triangle itself has symbolical meanings, whose bearings seem less controversial.



*Fig.1 floor map of Sant'Ivo, Albertina, Vienna*

Evidence of the second interpretation cannot be found on the several drawings by Borromini, but was instead drawn later, in 1720, by Sebastiano Giannini, pointing out its instrumental use for heraldic purposes, to the end of representing the bee of the Barberini family; we remark that the heraldic bee in itself could stem from either construction. We do not discuss further the implications of the salomonic symbolism; it could well have been an important principle of organization, but its structure is not reflected in the symmetry structure of sant'Ivo, in any of the three dimensions, which can be readily be accounted for without it.

In this paper we discuss the mathematical difference between an equilateral triangle and a six-pointed star, and argue that the entire building of Sant'Ivo is an equilateral triangle, not only at the ground level, but also up well into the dome, to a certain height, which will be interesting investigating per se. This observation yields a further question, far more sophisticated, concerning the shape changes that take place in the vertical dimension, in the unfolding of the dome. The question is strictly three-dimensional, as it regards changes in the sign of curvature of surfaces. We have not found reference to this problem in the scholarly literature about Sant'Ivo, while it is one of the aspects that strike the visitor as he gazes upward.

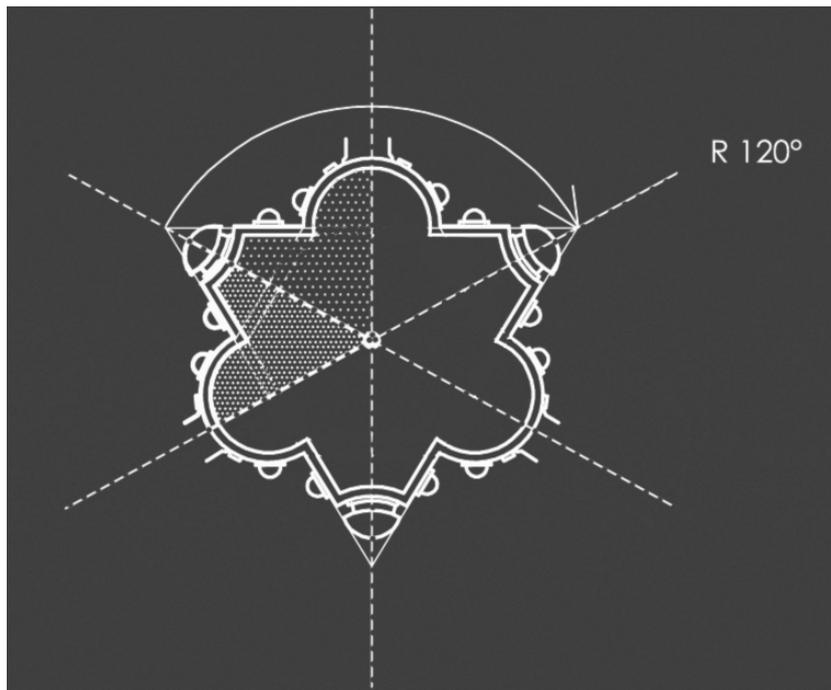
## 2 The floor plan and its symmetries unfolding in the walls

Today for mathematicians and geometry is characterized by its invariance under a group of transformations. A symmetric figure is invariant under rigid motions of the plane. This seemingly abstract treatment with regard to a built artifact, allows in fact to modeling the perception of a spectator, identifying symmetries in the invariance of his observations under his own motions in the space.

The group of symmetries of a regular polygon is called dihedral, and consists of rotations in a plane and mirrors.

We call “equilateral triangle” any structure, which is invariant under the dihedral group  $D_{2,3}$  generated by a rotation of  $2/3\pi$ , and a mirror along an axis cutting this angle in half. The group has 6 elements, i.e. the cyclic group of rotations of order 3, and three mirrors. Obviously a classically drawn equilateral triangle is invariant under such isometries of the plane.

We call “regular hexagon” any structure, which is invariant under the dihedral group  $D_{2,6}$  generated by a rotation of  $1/3\pi$ , and a mirror along an axis cutting this angle in half. The group has 12 elements, i.e. the cyclic group of rotations of order 6, and six mirrors. In particular a six-pointed star is a hexagon from the point of view of symmetric properties, as the same symmetry group characterizes it.



*Fig.2 analysis of the symmetry group of the plan.*

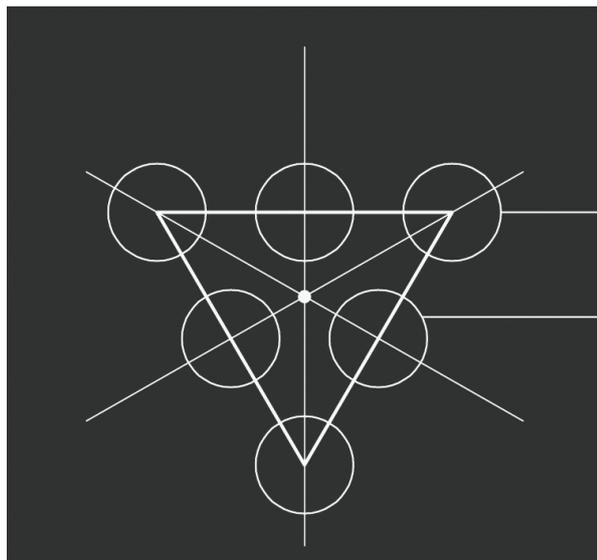
In fig. 2 we report a graphic analysis of the plan in fig.1. A symmetry analysis of the plan certainly yields those of an equilateral triangle, being invariant under rotations of multiples of  $2/3\pi$ , but not under a rotation of  $1/3\pi$ . In Fig.2 we report the fundamental domain of the group, dotted. Sparse dotting corresponds to the minimal area that can be rotated, and more dots correspond to the fundamental domain of the geometry, i.e. the smallest area that can regenerate the entire motive under rotations and mirrors  $D_{2,3}$ .



*Fig.3 the walls enclosing the spectator*

Coming to the three-dimensional aspect of the church, this symmetry is readily perceived by the spectator, not on the floor plan, but more naturally on the walls around him. In fig.3 an unfolding of what a spectator sees while rotating on himself. There are obviously three (and not six) congruent pieces of architecture, seen successively as the spectator rotates of an angle of  $2/3\pi$ ,  $120^\circ$ . In each piece, axis of mirror symmetries can be perceived; there appears to be six such axis, because we are really seeing in the picture their intersections on the surrounding walls: they obviously are paired on lines coming through the center of the plan.

As a spectator is surrounded by such walls, the feature that strikes the perception is that of the alternating concavity and convexity of the walls; this is also the feature that makes the repetition really three-folded. There exist a number of churches with six-fold symmetry, and in this case the corresponding niches are all concave. This concave-convex feature is typical of Borromini. On the plan, it is easily designed by the use of a compass as shown in fig.4; the compass has an opening of a third of the triangle's side, kept the for either the outward and the inward niches. The compass is pointed in the three vertices of the triangle for the convex niches, and in the midpoints of its sides for the concave ones. This is compatible with the traces of the points of the compass in the autograph in Albertina, as pointed out by prof. Gargano.



*Fig.4 How to design the niches with a compass.*

### 3 From the walls into the dome, a problem.

Once the spectator's eye is on the walls, it goes up into the dome. The dome rises from the three-folded symmetry, and continues it. Now, a dome is a surface of positive curvature (i.e., roughly speaking, both principal curvatures bend toward the same side of the surface); this definitely has symbolic unchangeable meanings, regardless of its possible plane sections: a surface of positive curvature mimics the celestial vault and also all containers. With respect to a central vertical axis, this dome obviously bends vertically toward the center, as all domes; the other curvature, that could be seen in sections parallel to the ground, is a continuation of concave and convex niches, i.e. of curves bending toward (concave) or away (convex) from the central axis. The continuation of the convex niches in a smooth way into the dome would yield a surface of negative curvature. One can also appreciate that as the dome unfolds upward the hexagonal symmetry is attained. We think the two problems: the correction of the negative curvature and its dissimulation, and attaining a hexagonal symmetry at the top (base of the lantern), are in fact the same problem. We would like the opinion on this of experts in history of science of constructions; at the moment it seems the windows opening in the dome play such a role of concealment, to the eye, but study of the structural elements is needed.



*Fig.5 Dome*

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in different public exhibits [3]. Alessandra Carlini, for assisting in obtaining copies of the Albertina document and the first graphic schemas of my analysis; and last but not least Elisa Conversano, for continuous discussions about archive sources and support.

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