Slovak University of Technology in Bratislava
Faculty of Mechanical Engineering
18th Conference on Applied Mathematics **APLIMAT 2019**

Proceedings

MATHEMATICS AND THE DOCUMENTATION OF INTANGIBLE CULTURAL HERITAGE: THE SOUND OF THE TREVI FOUNTAIN IN ROME

MAGRONE Paola (IT), TEDESCHINI LALLI Laura (IT)

Abstract. The highly complex sound of water is part of the cultural heritage of Piazza Fontana di Trevi. The spectral and temporal complexity of the signal defies classical methods, also due to the spatial organization of the fountain. We report on the sound on the larger scale of the piazza and its surroundings, by reviewing instrumental, perceptive and numerical simulations surveys. Trevi is a "mostra d'acqua": a fountain at the end of aqueducts exhibiting visually and auditively the richness of their waters.

Keywords: Complex sounds, soundscape, Fontana di Trevi, cultural heritage, water sound, perceptive assessment, ray-tracing, complexity.

Mathematics Subject Classification: 00A67, 65C20, 68U20

1 Introduction

Fontana di Trevi is one of the most famous symbols of Rome: recently movies and traditional songs contributed to increase its fame.

In Rome the big fountains placed at the end of the famous Roman aqueducts, were called "exhibits", since they exhibited the richness of water carried by the aqueduct; they often were monumental fountains. The fountain of Trevi is such a "*mostra d'acqua*". We recall the history of the fountain bearing two questions in mind: how long that sound has characterized the neighborhood, and how long the fountain has been organized in three recognizable groups of jets. Historical sources ([3] and [12], cited by D'Onofrio) report that since VIII century the fountain was in the place it is today, and that the aqueduct was almost always in operation. We can then affirm that the sound of water has characterized the small square for centuries; in this sense it is even mythological, it is an ancient sound, forming the soundscape part of the heritage of the quarter.

We are interested in the documentation and perception of this sound, in the piazza and in the surrounding alleys. The documentation of sounds in urban spaces is technically difficult; interesting from the spatial point of view, and not completely understood from the physical point of view. We think this is due to the complexity of the signals. Even numerical simulation of its spatial diffusion poses interesting questions. In this article we report about perceptive surveys and instrumental measurements, performed in the piazza ([5, 9]) addressing the problem of the recognizability of the sound of the water. The assessment of the quality of the sound of fountains, elusive as it might be, is quite important for soundscape studies, and for its applications in urban planning. With this respect its “pleasantness” has been carefully studied in [9].

The paper is organized as follows: Section 2 is devoted to a review of our perceptive surveys; Section 3 contains the numerical simulation of the propagation of a wave front, using a geometric model of Piazza Fontana di Trevi. In Section 4 we recall the history of the fountain, focusing on how long the area of Fontana di Trevi has been characterized by the sound of water and how long it was organized in three main sites.

Fig. 1. Fontana di Trevi, 2018. Source: Elle online magazine.

2 The sound of water in the Piazza and in the alleys: perceptive surveys, acoustic levels.

Not many romans, and not even the foreigners, notice the sound of the fountain. This is *per se* an interesting perceptive issue. In fact, the sound is there, and it is so important as to spoil all videos shot by an unaware shooter. In the movie by Federico Fellini, La Dolce Vita, the most famous scene takes place around and then literally *inside* Fontana di Trevi. Anita Eckberg (Silvia) must shout to be heard by Marcello Mastroianni (Marcello), even if he is standing quite close to her, because the sound of the fountain fills the scene, and this is faithful to the actual sound. People living nearby are in fact annoyed by the sound.

The human ear has the ability to recognize, and often separate sounds in a conglomerate. We have exploited this ability, in perceptive surveys. In previous papers the problem of the documentation of a sound which typifies an urban space is addressed: in this particular case, the sound of water marks the piazza and the surrounding alleys of Fontana di Trevi; we report here some of those results and considerations [5, 10]. The focus was on the recognizability of the sound of water, and on sound as an orientation factor. After performing perceptive surveys, some measurements of intensity levels were done, with a sonogram, together with Gerhard Mueller. Those measurements were accurately spatialized, as well as the perceptive surveys. We call this spatializations “earpoints”.

A team of graduate students of architecture performed the perceptive survey during day hours and at night, then reported the results on two maps (Figure 2). The darker the blue colour, the stronger was the perception of the sound of water, with respect to all other sounds: people chattering and shouting, the noise of air conditioners etc. For specific details, also on the method of collective surveying, see [11]. Looking at the two maps, it is very clear that at night the sound of the fountain can be heard from a distance, well before seeing it: the night surveys were done during winter, when the piazza is almost desert. A first nonobvious spatial observation is that to a perceiver standing inside the piazza masking of sounds happen in somewhat unexpected fashion. People's chattering in day-time does not cover the sound of the water. On the contrary, mechanical sounds, like the noise of fan of air conditioners, cover completely the sound of water when the perceiver stands very close to their source.

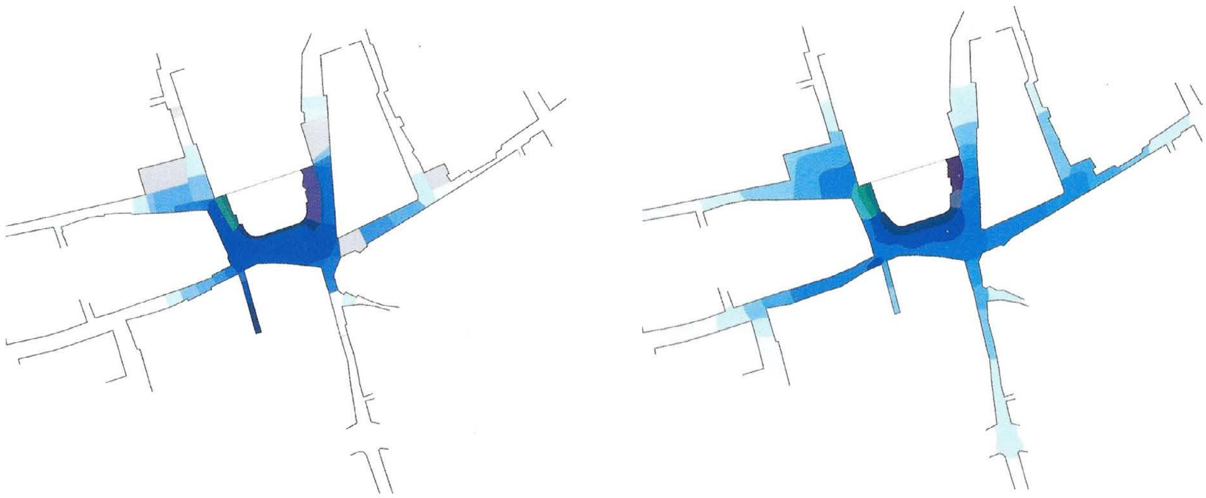

Fig. 2. Map of the perceptive survey performed in 2005 and reported in [5, 10]. Left: day survey, right: night survey. The shades of blue show the recognizability of the sound of water. Darker means that the sound is perceived more clearly. Reproduced from [10] with permission by authors.

From the two maps in Figure 2, it can be observed that when walking away from the fountain, the sound diminishes sometimes in a regular way, sometimes in a sudden way, and sometimes it remains constant for quite long sections. The sound of a fountain is strictly a complex one, in this case the fountain has three different jets of water, in a small square such that of Trevi; all this generates nontrivial interactions between sound and space.

In [10] it is discussed that some narrow streets seem to act as waveguides, and some narrow and short alleys produce early reflections. All in all, probably due to the complexity of the sound, Piazza Fontana di Trevi appears to be a gymnasium for studying psycho-acoustic phenomena [6].

3 A ray-tracing simulation

In this section we show a numerical simulation. We started with a 3d geometric model of Piazza Fontana di Trevi and its buildings. The data for the 3d model, including height of buildings were taken from Google maps. The aim of the simulation is to visualize the reflections of sound, modelled as a discrete system of particles spreading from a source, on the surface of a sphere, initially small: this small sphere is a mathematical model for a point-shaped sound source. Treating particles instead of waves, allows fast convergence and is quite reliable for reflections. In presence of no obstacles, density of particles on the surface of the enlarging sphere obviously diminishes with $1/r^2$, which is the well-known law for intensity of sound in open space. Under reflection, though, interesting features of overlapping paths can be observed, accurately simulating the organization of urban space (such spaces are called semi-reverberant in some technical literature).

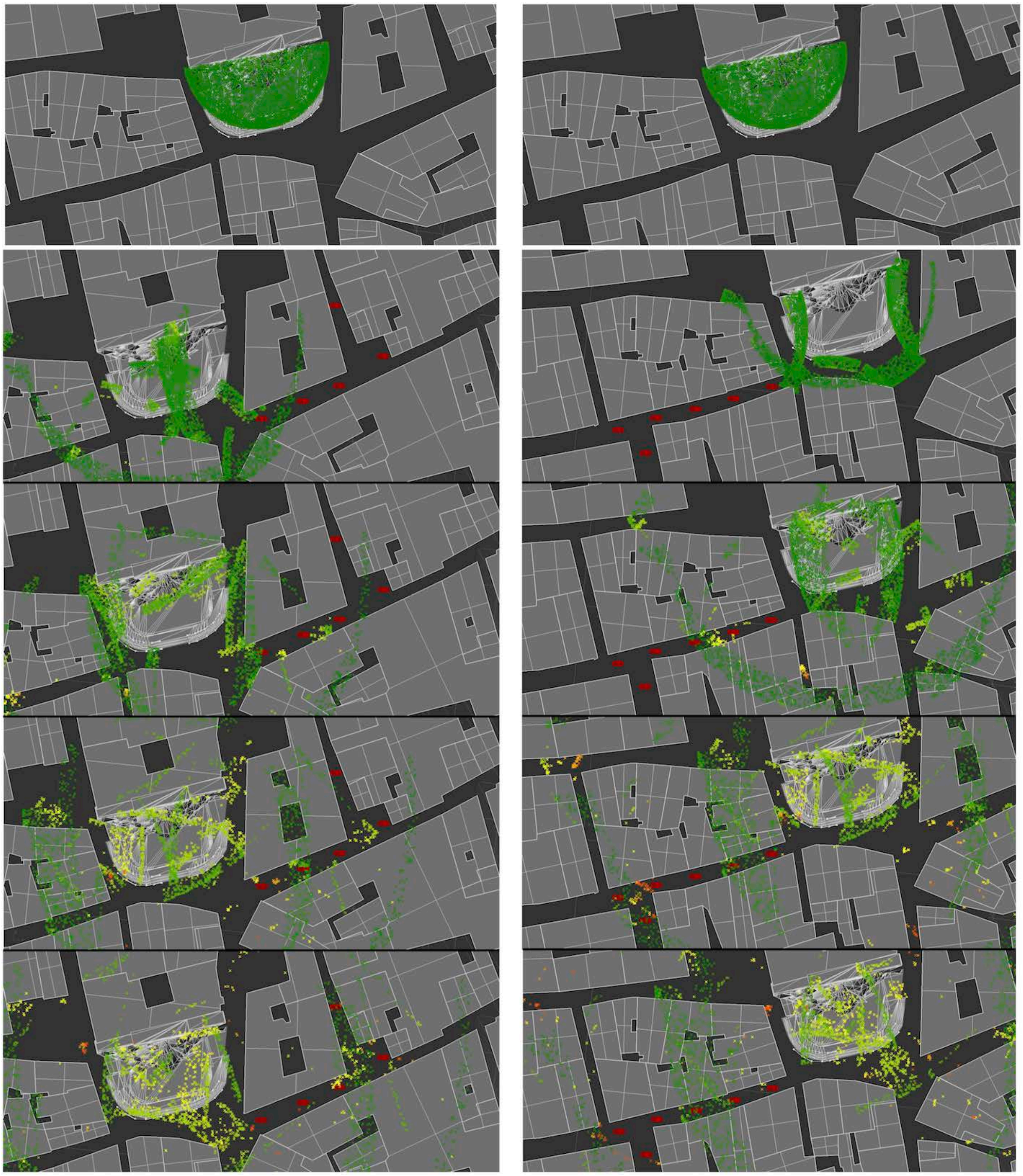

Fig. 3. Piazza Fontana di Trevi, ray tracing simulation, frames taken from 16 to 85ms. The green spherical sound wave widens in the space. Upon reflections, on obstacles, overlapping occurs. Notice at time 80ms ($8x10^{-2}$ sec) sound is trapped in the piazza. Reproduced from [5] with permission by authors.

On the other side, a particle model cannot deal with diffractions around obstacles, since each particle does not act itself as a wave front, and also because effects of different frequencies are ignored. Particle models in ray tracing are important: numerical tools, even sophisticated, used for room acoustics can be hardly used in urban spaces, for several reasons.

A few of these reasons are a matter of spatial scale, and the fact that urban spaces are neither really open nor really closed. Boundary conditions for the wave diffusion are a non-trivial issue: in the vertical direction the space has a “free boundary”; horizontally the conditions can be very complicated, both from a geometrical point of view (angles and singularities) and also in consideration of the different materials obstacles can be made of.

The numerical simulation was performed by architect Lorenzo Pierini with the software BAAT ([1]) and was published in [5] together with other raytracing models. The numerical piazza is made to lay inside a “virtual cube” of side 600 meters, whose top side is made completely absorbent, to simulate dispersion in the vertical direction. An appropriate noise reduction coefficient is given to building and streets, to simulate the reflections of sound onto these obstacles.

The actual fountain carries several jets, organized around three main sites; as a first simplification, the numerical model simulates only one, spreading from a central position: in this case the “source” generates 5000 particles, and is placed against the wall, close to the floor.

The frames in Figure 3 show the situation a few milliseconds after the beginning. Notice the first frame (both in left and right figures) shows clearly the portion of sphere; the second ones show the propagation of the wave fronts, which appear as arcs of particles. The intensity of sound is visualized by the density of particles; the color represents the energy: at the beginning particles are green (full of energy) and progressively they meet obstacles and loose energy (light green, then yellow and finally red).

In Figure 3 left, a bunch of particles can be seen trapped in the alley marked with red dots; the perceptive survey showed that sound of water was distinguishable in this narrow street (see Fig. 2).

Another effect was noticed in the alley “Vicolo del Forno”, the short and very narrow street at the bottom of the piazza (Figure 4). The perception of the sound of water is constant inside the alley. This could be due to precedence effect ([2]), a psychoacoustic effect: the perceivers, blindfolded, could not tell where the fountain was, once inside the alley. The ray-tracing shows a packet of particles trapped in the alley.

Also the measurements taken by a sonometer and reported in [5] confirm the effect in the alley. We have only looked and documented the differentiation into three nozzles. Changes due to the restoration, even in recent times, of the basins of collection are to be studied, as well as the insertion of new materials.

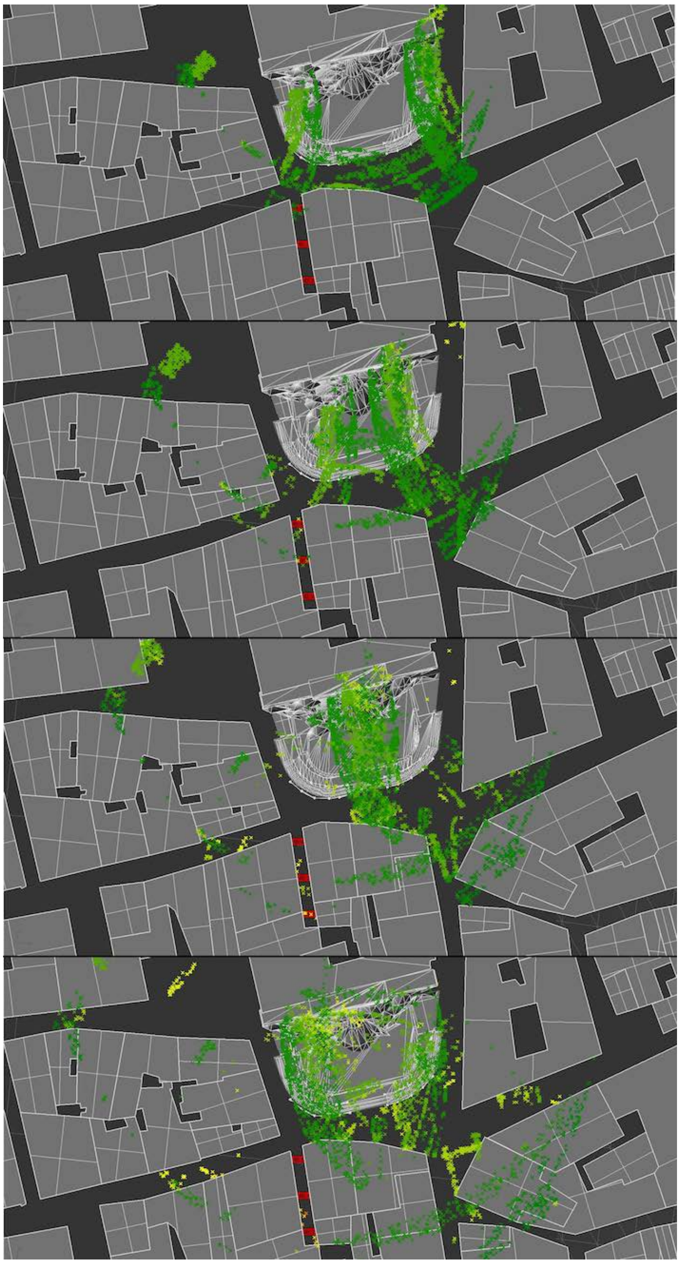

Fig. 4. Piazza Fontana di Trevi, ray tracing simulation showing the alley “Vicolo del Forno”, bottom left, where the red particles are visible. Frames taken from 29 to 50 ms. The alley is dead end, the sound is trapped. Reproduced from [5] with permission by authors.

3.1 Why a particle simulation: the complexity of the sound and configuration

All roman “water exhibits” have complex sounds, and certainly that of Fontana di Trevi is highly complex. It is complex because it is powerful, and because it is characterized by many different frequencies, and also by many impulsive phenomena. This means that its documentation is difficult: the recording equipment, even the sophisticated ones, are calibrated on space scales and frequency scales that do not necessarily take into account all the requirements that we would like to document. So first it will be necessary to inquire, in the future, on what are these requirements; a theoretical framework should be provided.

The diffusion of sound in air, i.e. waves in space, regards fluid dynamics. The mathematics necessary to describe the diffusion of a sound in a space like that of piazza Fontana di Trevi is an object of research at the moment. The space affected by the diffusion of the wave that we would like to study, sees buildings and streets as boundary conditions: we know that the sound of Fontana di Trevi spreads in the nearby streets, and we are in fact very much interested in what happens in the alleys. We therefore necessarily have to deal with boundary conditions with plenty of singularities. The simulation and integration of a wave requires integration in the space variable, from the source to the whole interested area; for Fontana di Trevi, the square (even if very small compared to the fountain itself) measures at least 30 meters.

The accuracy of numerical integration depends on the wavelength. And in any case numerical integration uses different methods and different rates, depending on the wavelength, which in turn is related to the frequency. The complexity of sound is reflected in the abundance of frequencies and the width of the area. This sound has many different frequencies, it is complex in the sense that the spectrum is extremely complex, some continuous bands of frequencies, and others that seem more isolated; there also seem to be resonances, perhaps due to the basin. A study on these aspects is underway. The particle simulation allows to study only the reflections and to study them quickly, accurately, and to follow the particles along their spatial path. The reflections also allow in turn at least to begin to describe the law $1 / r^2$. As we just saw, a raytracing also describes effects in the spaces that the technicians call semi-riverberants.

4 The *Vergine* Acqueduct and the building of the fountain

The present aspect of Fontana di Trevi was designed by architect Nicola Salvi (1697-1751). The fountain is fed by the *Acqua Vergine*, the water carried by the *Vergine* Aqueduct. This aqueduct was built by architect Agrippa, in 19 BC, during the reign of emperor Augusto; its sources are located in the South East of Rome, between the streets Tiburtina and Collatina [3, 8]. The source lies in a place called Trebium, from which the name Trevi might possibly be derived.

Fig. 5. The façade of the fountain, 1453. The earliest known documentation. It shows already three different and powerful cascades of water.

The route of this aqueduct is a little over 20 km long, smaller if compared to other very long aqueducts that were built by the Romans. A good part of its route is located underground. For these reasons the aqueduct, unlike many others, remained in operation throughout the Middle Ages. The name *Acqua Vergine* comes from the belief that a young woman pointed out the springs to a group of soldiers who were looking for water and they found a huge source in the point she suggested.

Literature [3, 12] reports the existence of a “Mostra d’acqua” of this aqueduct in the position occupied today by Fontana di Trevi at least starting from the 8th century. A first restoration was ordered by Pope Nicolò V (1397-1455), as can be read on the plaque of the figure (Fig. 5) of 1453. From successive original documents by Giacomo Dellaporta (1532-1602), we can have a precise idea of the measures of the fountain at that time: the basin was 13.50 m long, 17 meters wide [3]; it descended from the square's floor by a staircase protected by 18 columns.

The fountain has always had a three-folded partition, as can be ascertained from all the available drawings and paintings (for example see Fig. 5 and 6 left). This partition affects the complexity of the sound itself. Pope Urban VIII (1623-44) commissioned Bernini to renovate the façade of the water exhibition of *Vergine* Aqueduct. The Pope allocated the money and gave him the permission to use the marbles of the beautiful tomb of Cecilia Metella on Appia Antica. The municipality opposed to the fact that the monument of Cecilia Metella would be disfigured, so the works were interrupted. Bernini managed anyway to give a new structure to the fountain, in particular he placed a circular central pedestal that should have had the function of supporting a sculpture, and a double tank, in anticipation of the strong water pressure coming from the restoration of the aqueduct.

Fig. 6. Left: map of Rome by Antonio Tempesta ,1593; right: map by Giuseppe Vasi, 1781.

Bernini also changed the orientation of the fountain, as it can be noticed comparing the maps in Figure 6: on the left, the map by Antonio Tempesta (1555-1630) dating 1593 shows the fountain (recognizable by the three jets) facing the Antonina Column, so also facing the Pantheon which does not appear in the picture. On the right map by Giuseppe Vasi (1710-1782), 1781, shows the fountain turned of 90 degrees.

In the requirements of a competition banned by the Accademia di San Luca in 1706 for the fountain, one of the conditions was the presence of three distinct jets and in almost all the drawings and projects showed in ([3,4]) the three-folded partition of the fountain is clearly visible.

The long story of Fontana di Trevi came to the present state with Pope Clement XII (1652-1740), around 1730. In 1730 a competition was banned; the Pope took the decision to authorize the proponents to invade the surrounding buildings, also providing demolitions if necessary. It is at this point that projects arrived, even more monumental.

The winning project was that of Nicola Salvi: being an "exhibition of water", Salvi gives great importance to the water itself, to really "exhibit" the water, and in his project the protagonists are rocks and waves. The works begun in 1732 and officially ended in 1762.

5 Acknowledgements

The authors thank the team of graduated students of the School of Architecture of Roma Tre University who performed the perceptive surveys of Fontana di Trevi in 2005. Claudia Ferrauto and Gianni Puri made the maps in Figure 2.

References

[1] BEGERON-MIRSKY, W., LIM, J., GULLIFORD, J., PATEL, A., Architectural acoustics for practitioners, in *Advances in Architectural Geometry*, pp 129-136, 2010.

[2] CANÉVET, G., La localizzazione uditiva dei suoni nello spazio, in Giuriati G. e Tedeschini Lalli L. Eds, Spazi sonori della musica, Palermo, L'Epos, 2010, pp. 53-78.

[3] D'ONOFRIO, C., Acque e fontane di Roma, Roma: Staderini Ed., 1977.

[4] LESTER COOKE, H. JR, *The Documents Relating to the Fountain of Trevi*, The Art Bullettin, Vol. 38, N. 3, pp149-173, 1956.

[5] MAGRONE, P., TEDESCHINI LALLI, L., The Listening of two Piazzas in Rome, Noise Mapping 2018; 5:86-103.

[6] MULLER, G., MOSER, M., Handbook of Engineering Acoustic, Springer Verlag 50 Berln, 2013.

[7] MURRAY SCHAFER, R., The Soundscape: Our Environment and the Tuning of the World, 1977 and 1993.

[8] NIBBY, A., Analisi storico-topografico-antiquaria della carta de' dintorni di Roma, tomo III, Roma: tipografia delle belle arti, 1837.

[9] RÅDSTEN EKMAN, M., LUNDÉN, P., NILSSON, M.E., Similarity and pleasantness assessments of water-fountain sounds recorded in urban public spaces, The Journal of the Acoustical Society of America 138, 3043, 2015.

[10] TEDESCHINI LALLI, L., *Mappatura sonica di un'area romana* in G. Giuriati e L. Tedeschini Lalli Eds, Spazi sonori della musica, Palermo, L'Epos, 2010; pp. 135-148.

[11] TEDESCHINI LALLI, L. Rilievi sonori in centro storico a Roma, Urbanistica 2014 153,137-141 – Sound surveys in the Historic District in Rome. In Urbanistica 153, 2014, pp. 141-143

[12] ZUCCHETTI, G., ed. *Il Chronicon di Benedetto Monaco di S. Andrea del Soratte e il Libellus de Imperatoria Potestate in Urbe Roma*, Roma: Tipografia del Senato, 1920.

Current address

Magrone Paola, Assistant Professor of Mathematical Analysis
Dipartimento di Architettura, Università Roma Tre
Via Madonna dei Monti 40, I-00184 Rome, Italy
E-mail: magrone@mat.uniroma3.it

Tedeschini Lalli Laura, Professor of Mathematical Physics
Dipartimento di Architettura, Università Roma Tre
Via Madonna dei Monti 40, I-00184 Rome, Italy
E-mail: tedeschi@mat.uniroma3.it