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A METALLIC 1928 GEODESIC DOME IN ROME

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Abstract. In Rome a metallic geodesic dome (1928) is still visible in its structure. The first geodesic dome ever built, in Jena (1926), is not visible any more, therefore the roman one is probably the eldest existing geodesic dome. Both the one in Jena and that in Rome are metallic grids and were planned to support a (spherical) screen for a planetarium. To argue it deserves the attribute "geodesic" we performed a first onfield topological survey concerning the vertex degrees.

Keywords: Geodesic domes, fullerenes, topology of grids, vertex degree.

Mathematics Subject Classification: Primary 00A67, 37J05

1 Introduction

It is often asked what can science do for other fields of human activities. A beautiful story going the other way is that of geodesic domes. An architectural invention went on to live a life of its own in today living science. The topological dual of a "geodesic dome", is today called a "fullerene", and keeps being object of research in various scientific fields. It is therefore interesting to inquire both how old the architectural idea of this space organization has been, and how much it went into unforseen fields. In this paper we address the first question, that of architectural earlier such structures. Before Buckminster Fuller [4], other metallic grid geodesic domes had been built and patented. Their geometry is different from that of Fuller, in that they do not proceed from the projection of a platonic solid onto a sphere. They retain the symmetry of a sphere, but as metallic grids they are not topologically equivalent to those of Fuller.

The interest in Buckminster Fuller's precise organization of space later inspired several studies. One of these later works, which was awarded a Nobel prize in Chemistry in 1996, [1], was the synthesis of a stable molecule by a particular arrangement of atoms given by the topological dual of the type of geodesic domes obtained by Fuller. The molecule was thus called "fullerene" in his honour, being the topological dual of the type geodesic domes obtained by Fuller. The importance for mathematicians is that a fullerene is given by a topological definition. The usual visualization model for a molecule is that of a polyhedron where the atoms are in the vertices, and the edges represent chemical valence. To study the interactions between the atoms whilst maintaining the polyhedron with straight segments of fixed length as edges means we are thinking, mechanically, of a "stick model", i.e., one where the distances between atoms are preserved. In fact interactions among atoms could be of a different nature;

one such interaction is modeled mechanically as “springs” [8]. Already this observation strongly underlines the importance of the topological (non metric) definition of the spatial organization of fullerenes. The importance of fullerenes, in fact is that it is an entire class of possible molecules, which keep being studied in their richness.

By following the topological clue of identifying vertices by their degree, we were able to establish the similarity between a 1926 dome built in Jena (Germany) and one built in 1928 in Rome (Italy). Both domes are intended for use as planetariums and are topologically different from Fuller's well known domes.

2 Geodesic architectural surfaces, the math

Given two points on a surface, we call “geodesic line” a line, which connects the two points, and is of minimum length on the surface. Therefore a geodesic line is a path minimizing locally distances on a surface. Such lines, evidently, depend on the surface they lie upon. For elementary introduction to geodesics, we recommend the first chapter of [14] and the beautiful booklet [12], illustrating geodesics on a sphere and on locally Euclidean surfaces, such as cylinders, without resorting to variational calculus. In the last two decades, a renewed interest has arisen, both from the structural engineering and architectural field, to exploit the elegance and structural efficiency of structural surfaces. This interest also prompted studies on different possible geodesic structures of built surfaces [3, 13].

We also define “geodesic” a triangular tessellation of a surface, in which some consecutive (possibly curved) sides of the triangles lie along geodesic curves of the surface. In this sense the triangulation is “intrinsic” to the surface, as much as geodesic curves are “intrinsic” to it. The surface, thus triangulated along geodesics, defines a lattice of points. If we join the lattice points by straight segments now, we have an approximation of the actual geodesic triangulation. These straight edges of the triangles can be designed as bars, and so can the joints with dihedral angles at each vertex; such triangulation with straight edges, with some requirement of regularity, is what is called a geodesic surface in contemporary architecture, and is used in general surfaces. The advantage of using triangular arrangements in architecture is that a triangle is hardly deformable, and always in a single plan.

Buckminster Fuller used the expression “geodesic dome” in his patent Building construction-US2682235A–29/06/1954 (Figure 1). His first patent of a geodesic dome concerned grid-shell domes. In his patents he speaks strictly and explicitly about spherical surfaces [1].

> “Of or pertaining to great circles of a sphere, or of arcs of such circles; as a geodesic line, hence a line which is a great circle or arc thereof; and as a geodesic pattern, hence a pattern created by the intersections of great circle lines or arcs, or their chords.”

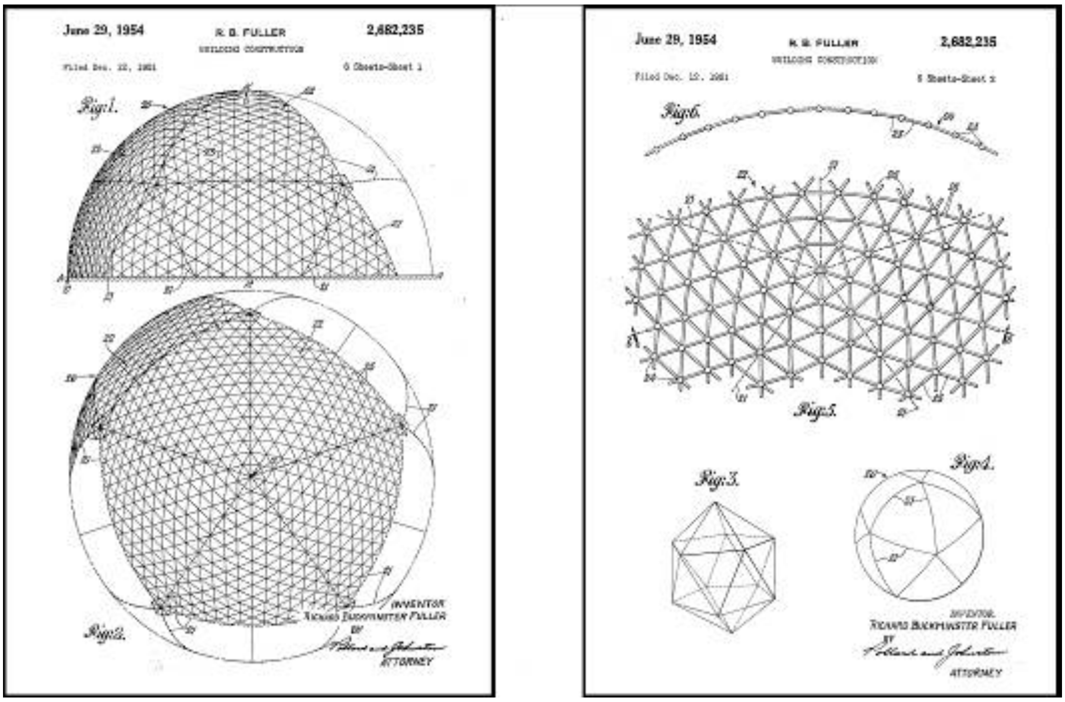

Figure 1. Buckminster Fuller's patent for Building construction (1954), US2682235-A, sheets 1 and 2.

He had probably discussed such geometries with other artists and with geometers at Black Mountain College in the 1940’s [10]. Subsequently, Fuller studied various possibilities. Important to our study is the fact that his patents and studies (unlike previous ones) deal with obtaining these geodesics by projecting a Platonic solid onto a circumscribed sphere, or the same procedure from semiregular polyhedra.

3 Earlier geodesic architectural surfaces

It is known that a spherical grid dome had been carried out successfully in Jena (Germany), culminating in the 1926 dome for the planetarium [2]. Already in 1922, Zeiss had experimented with the design of structures and screens for the projection of starry skies in a hemispherical hall [9]. The experiments were undertaken on commission from the Deutsches Museum of Munich, and realized in Jena, the town where Zeiss had its headquarters. Walther Bauersfeld and Franz Dischinger (of the firm of Dykerhoff & Wydmann) built the dome of diameter 16 meters. The Jena metallic structure was intended to be, and in fact was, the inner reinforcement for the thin shell of reinforced concrete that constituted the surface for the Planetarium projections. In this sense, such a dome definitely needs to be as spherical as possible, which presents a challenge for its construction. A few years later, in 1928 another Planetarium was built in Rome, Italy. Its inner structure was again a metallic triangulation of a sphere. There is less information on this dome than that of Jena. On the other hand, the dome is still on place. We will here address the geometric structure of the metallic grid-shell.

3.1 The Planetarium of 1928 in Rome

The geodesic dome of the first Planetarium in Rome has a diameter of about 19 meters. The metallic structure, still visible, used to serve as the supporting frame for a spherical cap that was required for the projections of the sky. This cap was built in wood with panels of reinforced canvas, a perishable material. Unfortunately, there is no photographic information about the screen cap, so it is not known how regular and smooth the actual spherical surface was. The metallic structure, instead, is visible in its original location, inside the Imperial artifacts preserved to this day. The result is a beautiful cohabitation of a dome in opus caementicium (Roman concrete) that pertains to the Imperial Baths (298-306 CE), and harbors the metallic structure of the twentieth century. Probably this is why the metallic one is so well maintained, unlike the one in Jena.

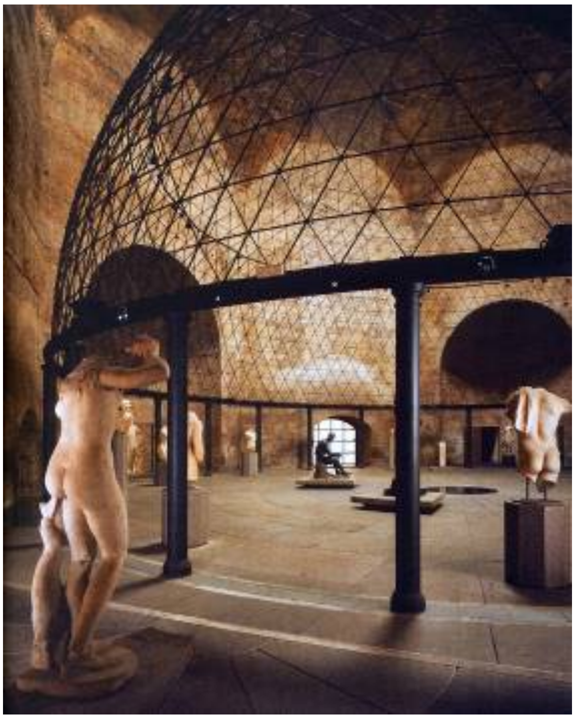

Figure 2. Planetarium in Rome (photo by Casabella. 1998, n. 654, March).

The Planetarium was inaugurated in 1928, and for its functioning, Germany endowed Italy with a powerful Zeiss projector, as part of the restitution for war damages from WWI [11]. Therefore the Jena Planetarium had already a kinship to the Rome one.

The kinship between the structure of the dome supporting the spherical screen in Rome, and that of Jena are evident [9]. The structure in Rome, built a few years later, afforded a larger diameter.

We refer to the appendix for the topological definitions and theorems. The geometry and the topology of the Planetarium seem very similar to what is known about that in Jena, from historical photographs. In particular, vertices have degree 5, 6, 7[1] (Figure 2). This seems peculiar to the modern eye accustomed with other geodesic domes. To the modern spectator, a degree 7 stands off. Modern geodesic domes are usually taken to the topological dual of a modern “fullerene”. It therefore has triangular faces, and only vertices of degree 5 or 6. The solid being topologically “simple”, it is easy to prove that there are exactly 12 vertices of degree 5, as we report in the appendix. Moreover to ensure symmetry, usually modern architectural geodesic domes are projections of regular or semi-regular polyhedra, none of which have vertexes of degree 7.

[1] Degree of a vertex, in a polyhedron, is the number of edges meeting at that vertex.

This is why, to a modern mathematical eye, a vertex of degree 7 stands off to the sight, and we proceeded to survey only these. We do not know why there are such vertices, but they are there, both in Rome and in Jena: we found the same kind of vertices in the old pictures for Jena, which are visible in the photos we shot at the Planetarium in Rome, and in the same spatial arrangement. In Figure 3 we indicate these vertices, which seem to form a pattern that is interesting per se.

Figure 3. Planetary Jena (historical photo from Greco C., Le prime cupole in cemento armato sottile, 1997, p.298) and Planetarium in Rome (photo by Casabella, 1998, n. 654, March). In the photos we indicate vertices of degree 7 and vertices of degree 5.

Structurally speaking, in both domes a triangular grid develops approximating a spherical shape, in a structure of surprising resistance, given the overall dimensions of the dome and the slenderness of the structural elements. In Rome the bars are made with metallic plates. They are then connected at their ends with iron disks, fastened with bolts [11].

3.2 A metallic twentieth century structure inside a Roman imperial bath hall

In Rome the Planetarium is located in a large hall of octagonal plan, already possessing a dome, the Octagonal Hall of the ancient Baths of Diocletian. At the time of the construction of the Planetarium, in the early 20th century, the Diocletian Baths hosted the Museo Nazionale Romano, as it does today. The choice of this location allowed the planetarium to be internal, shielded from weather, without having to build a large construction to house it. The Planetarium was positioned in the Octagonal Hall, in the northwest wing of the Baths of Diocletian (whose construction began in 298 CE and inaugurated in 396 CE). The Baths were part of the Western Gymnasium; its function is not completely clear. It probably carried a fountain in the center [11]. Recent archaeological results [7] assign it to a secondary *Frigidarium;* in Roman Baths, hot baths were followed by successive spaces to gradually cool off, *Calidarium*, *Tepidarium* and *Frigidarium*. The *Frigidarium* was often circular.

A dome is a dome…: a dome can accommodate more domes. In this case, the visitor today faces a large Roman Hall with a slender, elegant metallic structure, and the possibility of a third dome, made out of screen, over which the sky would be projected.

The spherical geometry enables the two domes to be in dialogue with one another. The brickwork umbrella dome, with a diameter of 22 meters, surrounds a concentric metallic grid, with a diameter of 19 meters, creating an embracing unitary central space.

Today the structure is visible thanks to restoration and refitting (1983-97), when it was decided to remove the hemispherical screen for the astronomical projections, while preserving skeletal structure, made of slender linear metallic elements. This hemispherical grid, with its large mesh, allows visual transparency, to the opaque brickwork cover, reinforcing the perception of the shared geometry between the two domes. The architect of the restoration was G. Bulian [5]. During this latter renovation Bulian decided to place some statues of the Roman Imperial period. The overall effect in such a wide emptiness reinforces a sense of spaciousness.

Appendix. How many pentagons to a fullerene?

As said in the introduction, "fullerenes" indicate a type of spatial arrangement for a molecule, and their name is a homage to Buckminster Fuller, by the field of Chemistry. The spatial arrangement, in turn spurned research in several scientific field.

We include here a definition for a fullerene, and a proof to the statement reported in section 3 about fullerenes. The statement itself is stunningly simple, its proof follows easily from Euler's formula for simple polyhedra, but is not often reported in literature, and is a basic fact about fullerenes.

Definition 1: A polyhedron is "*Simple*" if it can be deformed into a sphere by a continuous transformation.

Definition 2: a "*fullerene*" is a topologically simple polyhedron whose faces are pentagonal or hexagonal, and all vertices are of degree 3.

Notice there is no restriction about regularity of faces or their quantities, or symmetries. In fact, this theorem is purely topological.

Theorem: *A fullerene contains (exactly) 12 pentagonal faces.*

Proof: Let V denote the number of vertices, E the number of edges, and F the number of faces of the polyhedron under study.

A fullerene is topologically simple, so Euler's formulas holds for it (see for instance [6]):

$$V - E + F = 2$$

Let us denote as F_5 the number of pentagonal faces, and as F_6 the number of hexagonal faces. We can then rewrite Euler formula specifically for a fullerene:

$$V - E + F_5 + F_6 = 2$$

We can count all edges counting all vertices and multiplying by the number of edges meeting in each vertex (the degree of the vertex). Keeping in mind that obviously each edge belongs to 2 vertices (its end-points), and by hypothesis 3 edges meet in each vertex, it means that counting vertices and multiplying by 3 we count each edge twice. Thus

$$3\,V = 2\,E$$

By the same token, we can count all edges by counting all faces and multiplying by the number of edges of each face. Keeping in mind that obviously each edge belongs to 2 faces; it means by counting each of them face by face, we count them twice; counting face by face, we know each pentagonal face has 5 edges, and each hexagonal face has 6 edges:

$$2\,E = 5\,F_5 + 6\,F_6$$

Substituting these identities into Euler's formula for a fullerene, written above, we get

$$6\,V - 6\,E + 6\,F_5 + 6\,F_6 = 12$$

$$2\,E - 6\,E + (5\,F_5 + 6\,F_6) + F_5 = 12$$
$$-2\,E + 2\,E + F_5 = 12$$
$$F_5 = 12$$

Which is what we wanted to prove.

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